A Postscript Technical Report on Gravitational Driving

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Abstract. The aim of this postscript is to broaden the understanding of the model described in reference [8], as well as the mathematical approach to the theory previously discussed by examining the junction between two manifolds. Present work also makes some comments about metamaterials in the theoretical context of warp drives engineering.

Key words: warp driving, junction, metamaterial.

Resumo. O objetivo deste pós-escrito é ampliar o entendimento do modelo descrito na referência [8], bem como a abordagem matemática da teoria discutida anteriormente, examinando a junção entre duas variedades. O presente trabalho também faz alguns comentários sobre metamateriais no contexto teórico da engenharia de *warp drives*.

Palavras-chave: *warp driving*, junção, metamaterial.

Prolog

This technical report was motivated by an observation made on the expression to provide coordinate-invariant measures for the total energy from the warped region of a warp drive space-time. That expression was described by Bobrick & Martire [3], in the general set of warp drive modelling tools, and I rewritten it in my article "Gravitational Driving: Geodesics Warped by Solitons of Acceleration"[8], in which I treated the distortions in a world line caused by a

certain type of solitary wave. The study was conducted within the perspective of the geodesic warping that could be expected after the application of some gravitational soliton-based driving technology. It was not my objective in the original article to discuss the referred expression. However, I believe that it is worth adding some observations regarding the aims of the work and its limitations.

Metamaterial correction for a warp drive space-time

In a recent message, a colleague physicist pointed out to me a correction to be made in an expression that establishes the total energy contained in the warped region of a warp drive bubble, this correction being due to the intervention of some hypothetical metamaterial originating the warped reconfiguration of space-time. After going through the draft sent, it seems that the author offers a suggestion to approach the energy problem with metamaterial resonances (at least that's what I understood from the few handwritten lines).

The original expression is

$$E = \int_{\mathcal{M}_{warp}} \left(-g^{00}\right)^{-1} T^{00} \sqrt{-g} d^3 x^i_{out}, \tag{1}$$

where $\left(-g^{00}\right)^{-1} T^{00}$ is the energy density. The proposed correction (in the exact form in which it was posted) is

$$E = \mathrm{Re}\left[\int_{\mathcal{M}_{warp}} \frac{\left(-g_{00}\right)^{-1} T_{00} \sqrt{-g}}{S} d^3 x\right].$$

As far as it can be learned from the comments, S is in fact the contraction of the magnetoelectric susceptibility tensor related to the metamaterial, and, assuming an isotropic material, we have

$$S = \hat{\mu}^{\eta\iota\kappa\lambda} \hat{\mu}_{\eta\iota\kappa\lambda} \rightarrow \left(\varepsilon^2 + \frac{1}{\mu^2}\right).$$

Evidently, as it was a very succinct note, the author establishes $\varepsilon = \varepsilon' + i\varepsilon''$ and $\mu = \mu' + i\mu''$, but he did not have enough textual space to expand the discussion to give further details about the logical-motivational chaining of the outlined reasoning, nor about the physical ties related to the considered metamaterial and the geometry of space-time. The expression posted seems to show

interesting features, but I would need to dispose some time to take a look at its content more accurately to try to discuss my ideas adding value from other considerations regarding the introduction of metamaterials. Anyway, it is necessary to see if that expression really brought anything new and consistent with the reasoning currently accepted. For now, I will make some brief considerations about metamaterials, going directly to the fundamental points of the theory I advocate.

How far we are from practical applications

To fabricate a localized gravitational distortion — a warp field — there are many puzzles to be solved. Not to mention the unobtainable amounts of energy required, shall we be able to control the warpdrive bubble from within, and, if so, shall we be able to deal with the potential causation paradoxes involved and other potential side-effects? Since these questions shall remain challenging us for a long time, I think we should continue to refine the theory before proceeding further in technology of metamaterials and others.

Metamaterials are manufactured composites of metals and polymers, engineered with the aim to engender phenomena that otherwise would not be feasible or very difficult to observe. Studies from Smolyaninov [7] show that, for subluminal speeds, the metric he introduces points to promising possibilities for warping.

All the preliminary reasoning necessary to consider the use of metamaterials was found in Maxwell's classical macroscopic theory, and, in the domain of space-time geometries, it now includes general relativity. Logically, if the geometry will be modified, it is expected that the metric reflects the influence of the causative metamaterial. From the macroscopic Maxwell equations, in a metamaterial such as one built from split ring resonators and endowed with significant magnetoelectric susceptibility, it is possible to set

$$\varepsilon = \mu = h^{-1/2} = \frac{1}{\sqrt{\frac{1}{n_\infty^2} - \frac{v_0^2}{c^2}\tilde{f}^2(\tilde{x})}},$$

where $\tilde{x} = x - v_0 t$ and n_∞ is a scaling constant. According to reference [7], this equality is related to the gravitational field written as a suitable modification of the original Alcubierre metric form in the rest frame of the warp bubble,

$$ds^2 = \left(\frac{1}{n_\infty^2} - \frac{v_0^2}{c^2}\tilde{f}^2(\tilde{x})\right)c^2dt^2 - d\tilde{x}^2 - 2v_0\tilde{f}(\tilde{x})d\tilde{x}dt - dy^2 - dz^2,$$

$$ds^2 = \frac{1}{\varepsilon^2}c^2dt^2 - 2v_0\tilde{f}^2(\tilde{x})d\tilde{x}dt - d\tilde{x}^2 - dy^2 - dz^2,$$

to which three-dimensional Maxwell equations are

$$\boldsymbol{D} = \frac{\boldsymbol{E}}{\sqrt{h}} + [\boldsymbol{Hg}]$$

and

$$\boldsymbol{B} = \frac{\boldsymbol{H}}{\sqrt{h}} + [\boldsymbol{gE}]\,,$$

with $h = g_{00}$ and $g_\alpha = -g_{0\alpha}/g_{00}$. Also, in a subluminal warp drive model based on the magnetoelectric effect, it must be valid the inequality

$$\frac{v_0}{c}\tilde{f}(\tilde{x}) \leqslant \frac{n_\infty - 1}{n_\infty^2}.$$

This result is well explained by Smolyaninov [7]; it means that, for $n_\infty > 1$ in a material medium, the warp drive model is thermodynamically stable at subluminal speeds, while for $n_\infty = 1$ (in vacuum) the warp drive is forbidden.

It is now known that electromagnetic metamaterials make it possible to simulate exotic space-time geometries, especially the Big-Bang singularity. So it is very interesting to lead studies on metamaterial-based models of warp drives, searching its limitations and new developments on the so-called "bi-anisotropic non-reciprocal magnetoelectric metamaterials". However, in present moment it seems premature to open an explicit discussion of how these metamaterials would be implicated in the subject of my previous work, both mathematically and physically, since the *desideratum* was simply to describe a world line warp caused by a hypothetical human-made sine-Gordon-type soliton, a single distortion in the space-time woof with no prior concerns about unreachable energies (by the way, it's also worth considering Füzfa's work about an experiment devised to create a localized gravitational distortion detectable in lab [1]). For future works, the implication of metamaterials in simulations to restructure the spacetime metric shall certainly be a productive approach. In the meantime, I would like to share more information on the theoretical content developed in reference [8].

Engineering and matching the metrics

Alcubierre drive and other warp drive proposals are sound ways to 'apparently' overcome the speed-of-light barrier [2][3][4][5], however far reaching for our current civilization. A solid ground from mainstream theoretical physics is now available to treat such topics as warp drives, but we do not have a clue about what energy sources might be engineered for spaceflight applications. Perhaps the solution lies in the interactions between matter and vacuum, or, as I propose, in the space-time expansion energy itself.

My research program has a two-fold objective: 1) to introduce a quantum space-time expansion model and 2) to formally associate this model with a solitonic warp drive design that implies the possibility of introducing a shape function in a less arbitrary way, since, ultimately, what Alcubierre metric (and others like it) describes is a manner of inducing a curvature of space-time similarly to what a solitary ocean wave does, being reasonable to assume that the space-time warp drive bubble is shapebounded by a soliton of gravitational acceleration. The quantum structure of space-time based on its expansion is discussed in reference [6], and is part of my main arguments in cosmology.

Although space-time is expanding at each point, in certain circumstances the opposite effect of contraction caused by massive bodies in its surroundings occurs. Regions where there is a conflict between expansion and contraction were called by me "G-closures"[6]. Thus, a warp drive space-time bubble is, in a way, an anthropic G-closure where the opposite ends in the direction of displacement are dynamically conflicting with each other. In other words, behind the bubble the expansion of space-time is greatly enhanced, on the same foot that it is inverted in front of the bubble[1]. Having in mind the complexity of such an object, the technology to engineering spacetimes is part of an area that is worth looking at. The anthropic solitons of acceleration I came up with, as a result of metric engineering by advanced technological devices, must be produced from the necessary matter to deploy the intrinsic energy of expansion within the quanta of space-time to control a high-expanding curvature zone behind the starship, and a high-shrinking curvature zone in front, both in equal amount so that the starvessel resides in a stable bubble, moving with space; but we have no idea how to do this!

Increasing the space-time expansion energy is not a difficult idea to assimilate. If we think of the ocean as a medium in constant motion, with waves breaking the blue-green surface, we can imagine a simple device stirring the waters and transferring more energy to the oscillations. We can thus increase the impact of the wave fronts on a bulkhead equipped to measure the amount of energy transferred during the shock of the wave crests. The difference is that the proper energy of the space-time quanta (such as the churning energy of an ocean wave pulse in its "ground state") is very small; in order to cause a significant warping effect, one can immediately think of implementing a resonance process on the fundamental frequency associated with the space-time quantum expansion energy, but an extraordinary energy (negative or not) at least of the order of $10^{-1}M_{\odot}v_s/c$ (considering more reasonable requirements) would be necessary for that warping effect. In view of this, for the time being, all we can do is advance the theory and accumulate as much useful knowledge as possible, so that, in the future, we can better understand the technology we speculate about today.

So, for simplicity, without properly discussing the drawbacks associated with negative energy requirements, causality violations due to "apparent" superluminal velocities, and the fact that we don't know to create the shape function to engineer the Alcubierre warp drive space-time, I will assume the Alcubierre metric and make some considerations equally valid for other metrics, preserving the condition that $f(r_s)$ tends to 1 for $r_{(s)} \approx 0$, having in mind that the warp bubble moves as a whole with respect to the flat space-time outside it.

[1] The main point of the theory is the "natural engine" of expansion and contraction of space-time to be explored by a hypothetical technology powerful enough to do so. Even Natario's model runs with a contraction in front of the starship, however, compensated by an expansion in the direction perpendicular to the starship's motion in order to conserve the volume. So, whatever the model of warp drive, I believe that the main ground is in this dialectic of space-time expansion/shrinking.

We can rewrite the original Alcubierre metric, $ds^2 = -c^2dt^2 + (dx - f(r_s)v_s dt)^2 + dy^2 + dz^2$, in the form

$$ds^2 = \left(-c^2 + f_{(r_s)}^2 v_s^2\right) dt^2 - 2f_{(r_s)}v_s dxdt + dx^2 + dy^2 + dz^2. \tag{2}$$

The world line equation deduced in the original article [8] provides — under certain very special conditions and by means of a convenient choice of indices — a relationship that allows the determination of non-diagonal metric tensor components (diagonal components are, arbitrarily, extracted from the terms $-2f_{(r_s)}v_s dxdt + dx^2 + dy^2 + dz^2$ of the Alcubierre metric presented in equation (2). Therefore, the new metric obtained represents, in a way, the "touch" of the world line that defines the warp-drive bubble boundary on the manifold determined by the Alcubierre metric. This is an unorthodox approach, however, sufficiently plausible to establish a matching between the two metrics considered from a geodesic warped by a soliton.

Suppose that, at a point on the warp-drive bubble, considering the speed of the bubble constant over a certain small period of time, we can adopt, at that point, a static metric as

$$ds^2 = -e^{2\phi}dt^2 + \xi_{ij}dx^i dx^j.$$

Thereby, we can write

$$\xi_{ij} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -2f_{(r_s)}v_s \end{bmatrix}.$$

Assuming 1) finite quantum intervals of space-time[2], 2) $i = j = k$, with l as a running index, and 3) the spatial metric relation[3] (without summation, so that we produce an ordered set of numbers labelled with the indices l and k)

$$\xi_{lk} = \delta_l^j \frac{2}{E} \frac{\mathrm{X}^i}{\mathrm{X}^k} \xi_{ij}, \tag{3}$$

[2] Finite intervals on the Planckian scale, however, expansion as small as desired; this is the main philosophical foundation of the theory, somewhat inspired by Lazare Carnot's "Metaphysics of Infinitesimal Calculus" with regard to the kind of thoughtful insight into the essence of things. The introduction of these finite intervals as the ultimate constituents of the world lines (the quanta of space-time) allows for analysis of transition between regions by jumps, for example, time-like geodesics that enter a different region from a static region, becoming space-like. The expectation value measure of the rate in which the invariant element evolves only in time mode was deduced from $\langle 0| g_{\mu\nu} d\langle x - \varepsilon\rangle_\mu d\langle x - \varepsilon\rangle_\eta |0\rangle = \Omega^2 \left\{-\left[1 - C(u)\right] du^2\right\}$, where u is a time function that corresponds to $1/H$ for time coordinate equal to 0 and to 0 for time coordinate equal to ∞, and $C(u)$ was defined from the retarded Green's functions of the massless minimally coupled and conformally coupled scalars, being freely evaluated non-perturbatively. A complete approach to this model is found in reference [6].

[3] Indeed, this expression is a transformation rule that establishes how the spatial metric ξ_{ij} originates the energy configuration of the warped region.

$$\delta_l^j \begin{cases} 0, \ if \ j = l \\ 1, \ if \ j \neq l \end{cases}, \tag{4}$$

yielded from the equation for a sine-Gordon-type soliton-warped geodesic [8],

$$\frac{1}{E}\frac{d}{ds}\left(\xi_{ij}\,\widetilde{\mathrm{X}}^j\right) + \frac{\partial\phi}{\partial \mathrm{X}^i}\tilde{t} - \frac{\partial\xi_{jk}}{\partial \mathrm{X}^i}\frac{\widetilde{\mathrm{X}}^j\widetilde{\mathrm{X}}^k}{2} + m^2\tilde{t}\sin\vartheta\frac{\partial\vartheta}{\partial \mathrm{X}^i} = 0, \tag{5}$$

we can write, for small values of ϑ (see equation (31) in reference [8]), a null-diagonal matrix for the warped region

$$\xi_{lk} = \begin{bmatrix} 0 & 2E^{-1} & 2E^{-1} & -4E^{-1}f_{(r_s)}v_s \\ 2E^{-1} & 0 & 2E^{-1} & -4E^{-1}f_{(r_s)}v_s \\ 2E^{-1} & 2E^{-1} & 0 & -4E^{-1}f_{(r_s)}v_s \\ 2E^{-1} & 2E^{-1} & 2E^{-1} & 0 \end{bmatrix}. \tag{6}$$

The index-renaming δ_l^j-matrix is an "inverted" Kronecker delta. The internal energy E of the X^i intervals is the proper energy of space-time expansion. The matrix ξ_{lk} can be interpreted as the object that describes the spatial deformation as a function of the expansion energy E (in the natural system, with $\hbar = 1$, units of time and length are $1/E$, whose conversions are respectively $1GeV^{-1} = 6.58 \times 10^{-25}s$ and $1GeV^{-1} = 0.197 \times 10^{-15}m$). In such conditions, to finally match the Alcubierre metric with the static metric it is enough to write

$$\phi = \frac{\ln\left(c^2 - f_{(r_s)}^2 v_s^2\right)}{2} \to \frac{\ln\left(c^2 - v_s^2\right)}{2}. \tag{7}$$

Once equation (5) delineates the boundary between the inside and the outside of the warp-drive bubble, it can be a starting point for a less arbitrary determination of the shape function. Note that in the junction, expression (7) gives

$$f_{(r_s)} = \frac{\sqrt{c^2 - e^{2\phi}}}{v_s},$$

so that, keeping natural units, matrix (6) becomes

$$\xi_{lk} = \begin{bmatrix} 0 & 2E^{-1} & 2E^{-1} & -4E^{-1}\sqrt{1 - e^{2\phi}} \\ 2E^{-1} & 0 & 2E^{-1} & -4E^{-1}\sqrt{1 - e^{2\phi}} \\ 2E^{-1} & 2E^{-1} & 0 & -4E^{-1}\sqrt{1 - e^{2\phi}} \\ 2E^{-1} & 2E^{-1} & 2E^{-1} & 0 \end{bmatrix}. \tag{8}$$

Since the X^i intervals are subject to stretching and contraction, the tensor ξ_{lk} was called "tidal tensor". The complete formal sequence, including the theory presented in reference [8], is summarized as follows:

1. The tidal tensor is defined as the metric warped by the accelerating soliton in terms of the intrinsic energy of stretching/contraction;

2. The warped geodesic equation (5) provides, under certain specific conditions, the expression of transforming static metric components into tidal metric components;
3. From the junction between the Alcubierre metric and the static metric it is obtained a shape function;
4. The shape function is replaced in the tidal tensor.

Last comments

Arthur Clarke was a visionary who lived to see many of his predictions fulfilled, leaving others for the future. Carl Sagan was an audacious scientist, open-minded yet rigorous. Both represent the spirit that should guide innovative thinking for generations to come. I have always believed that even the topics of physics that are farthest from immediate applicability are subjects that add value to pure and simple knowledge insofar as they excite reason and show us the effort necessary to understand the world. It has been recognized that the study of warp drives leads us to identify what constraints established by the laws of physics would impose limitations on an arbitrarily advanced technological civilization, also serving as an innovative and exciting way to teach general relativity to young learners [9]. Furthermore, going beyond the interest of physicists dedicated to extraordinary topics, exotic field propulsion is a theme that has come to the fore, and a serious research issue for both NASA Breakthrough Propulsion Program and British Aerospace Project Greenglow, a fact that shows the relevance of studies directed to the topic.

With innovative thinking, I believe that, cleverly "subverting" (not breaking!) physics, bypassing the exoticism demanded by the first warp drive design, applying Füzfa's empirical results in association with the afore described formalism, we can take a step further in improving the theory. In principle, the solitonic bubble toy model, as an anthropic G-closure, would be generated in such way that internal electromagnetic field emitters in the starship would act to locally curve space-time, aft-expanding and fore-contracting the cosmic woof in the direction of displacement. The big challenge shall be to find ways to control the signatures of the predicted contractions and expansions of space-time and manipulate its intrinsic energy to produce very strong curvatures. I can't imagine what degree of technological and scientific advance it would take to accomplish such a feat.

Foremost, the ideas discussed here are no longer science fiction. Humanity shall certainly leave Earth sooner or later; it's a matter of survival. How far we shall go, no one knows[4]. However, if we want to get to the nearest star systems,

[4]Did you ever hear of the complexity of keeping humans in space for long periods? Even restricted to the solar system this shall be a serious problem if we only think about conventional propulsion engines, not to mention the slowness of the constant transport operations of equipments and materials necessary for colonization. Exotic engines associated to advanced unities of shortrange propulsion for precision movements would be of great value to take trips to Saturn's moons!

like that of the red dwarf TRAPPIST-1, with its three or four earth-sized planets in the Goldilocks zone, around 39 light-years from the Sun in Aquarius constellation, we shall need some very uncommon engineering. Meanwhile, a little caution wouldn't hurt. We are living in a phase of free lucubration, and, in a way, also of excesses, taking the discussion even to the field of extra dimensions[9]. In these moments of high intellectual tension, it's easy to move from science to intelligent design. Unlike extra dimensions, warp drives are testable constructs belonging to a falsifiable theory. The development of metamaterials, Füzfa's experiments, and Smolyaninov's results provide concrete empirical foundations to carry the theory forward, although still in the beginning.

Personally, I don't see much practical future in extra dimensions at the moment, and frankly, I would be happy and surprised if they were ever to become an everyday reality in science, since I have taken them into account in my theories for a long time. The extra dimensions associated with string theory constitute a kind of postmodern framework, closer to intelligent design than to science as we understand it, since, at least for the time being, one cannot speak of a real revolution in physics based on strings. I've been thinking over and over again of Occam's Razor, and I wonder if we're getting too complicated for so little clarification. I also remember Husserl, for whom the unveiling that makes possible the frenetic mathematization of science is also the reason for its obscurity. It must be remembered that we are physicists first of all (not mathematicians); we need to preserve physics with its real-world power of signification while maintaining objectivity and clarity. In the end, I find it difficult to resolve the fundamental issues by increasing the amount of unobservable dimensions; we have enough physics, as long as we know how to rethink the relationships between objects and systems "out of the box".

Lastly, I emphasize that the philosophy applied to physics had a great influence on the research in curse. In fact, I don't see how physics can progress significantly without this exercise. The relative stagnation of the current physics is partly due to the physicists' frequent disinterest in philosophy, perhaps due to absence of self-criticism regarding our limitations, perhaps due to incomplete intellectual preparation for science. For, looking at the portrait of giants like Bohr, Heisenberg and Jordan, on one side, and Bunge, Popper and Russel on the other, it is unlikely that we will see in this century such an intense approximation between physics and philosophy as we saw in the 20th century, especially until the 60s. There is, however, some hope. Although few, eminent physicists like Baez, Rovelli and Smolin seem to recognize the importance of philosophy not only playing the role of criticizing what already exists, but with a constructive mission with physics. It is up to philosophers to take an interest in physics as well.

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