CALIBRE — Review of the original 2011 preprint
November 2022

Advanced predator-prey modelling for work and employment scenarios

Brazil in focus

Nilo Serpa · José Roberto Steiner

Received: 25 Jul 2022 / Accepted: 30 Jul 2022 / Published: 06 Nov 2022

Abstract The broad application range of the predator-prey modelling enabled the conjecture that it may be applied to represent the dynamics of the work-employment system in Brazil. The simulations performed showed more chaotic dynamics at the beginning of the time series, tending to less perturbed states, as time goes by, due to public policies and hidden intrinsic system features. Basic Lotka-Volterra approach was revised and adapted to the reality of the study. The aim of this article is to show that the work-employment system in Brazil admits a predator-prey modelling, providing decision makers with generalized theoretical elements that allow to a more accurate and deterministic understanding of the relations between employer and employee in that country.

Keywords Predator-prey model · Chaotic dynamics · Work-employment system · Perturbed states · Evolution

Resumo: O amplo campo de aplicação da modelagem predador-presa permitiu conjecturar que esta pode ser usada para representar a dinâmica do trabalho-emprego no Brasil. As simulações realizadas exibiram uma dinâmica mais caótica no início da série temporal, tendendo a estados menos perturbados, com o passar do tempo, devido às políticas públicas e às características intrínsecas do sistema. A abordagem básica de Lotka-Volterra foi revisada e adaptada à realidade do estudo. O objetivo deste artigo é mostrar que o trabalho-emprego no Brasil admite uma modelagem predador-presa,

Nilo Serpa
Pesquisador, Centro Universitário ICESP, Brasília, Brasil
Tel.: +55(61)99234-0308
E-mail: nilo.serpa@icesp.edu.br

José Roberto Steiner
Cientista de Dados, Grupo Boticário, Brasília, Brasil

fornecendo aos tomadores de decisão elementos teóricos generalizados que permitem uma compreensão mais precisa e determinista das relações entre empregador e empregado no país.

Palavras-chave: Modelo predador-presa; dinâmica caótica; trabalho-emprego; estados perturbados; evolução

"Economies possess general ecosystem properties, such as dynamism, evolution, integrity, stability and resilience. Economies are inextricably embedded in larger natural ecosystems, and exchange flows of materials and energy with natural systems".

S. Farber & D. Bradley

1 Prologue

This article was originally divulged in March 2011 as a preprint in the Archives of Cornell University under the tag arXiv:1102.4528v3. It is a mathematically dense work that, most likely for neo-liberal ideological objections, was rejected in one specialized journal. At the time, the reviewer's argument was an inquiring speech about why the prey should be the worker and not the employer, insinuating the idea of the worker as the cause of all socio-economic imbalances. In particular, I believe that it is not the role of the reviewer to judge the merits of an article based on their personal beliefs. However, the rejection created the opportunity to add some detail on the almost obvious reasons for the assumption of the worker as the prey. Eleven years later, after considerable receptivity among economists and econophysicists, CALIBRE now publishes a revised version with some minor corrections. Although the work has not continued for restrictions on the access to the relevant databases, the research premises stay valid, and the robust mathematical model developed remains available for further simulations.

Nilo Serpa

2 Background

The concept of equilibrium in the predator-prey population dynamics has its origins in the works of Lotka (1) and Volterra (2), making the base of several theoretical models of interaction among species, with applications in the context of ecological systems and wildlife management ((4); (14); (15)), including fluctuating environments (16). Particularly, Blasius and Stone (4) were very benefited by previous works on phase synchronization phenomena in coupled chaotic systems (8). Also, there are generalized approaches of the Lotka-Volterra model (17). There is special interest on the problem of the growth of two populations conflicting with one another, known as "the problem of Volterra" (9). In fact, according to Guerraggio and Paoloni (3), however the American physicist and chemist Lotka had constructed a model formally analogous to that of Volterra, the interests and methodologies were quite different and, until the publication referred to his inaugural lecture of 1901, Volterra didn't know the work of Lotka:

Volterra zitiert Lotka nicht, da er dessen Arbeiten zu diesem Zeitpunkt nicht kennt. Später hat Volterra keine Schwierigkeit, die Verdienste des Kollegen anzuerkennen, auch wenn er einige Unterschiede hervorhebt: "Ich bedauere, das ich in der besagten Abhandlung die interessante Arbeit des Dr. Lotka nicht habe erwähnen können, die verschiedene Anwendungen der Mathematik auf chemische und biologische Fragen enthält" (German traduction from the original in Italian by Manfred Stern)[1].

The generality of predator-prey models becomes possible to abstract them from their early ecological roots, and, by analogy, to apply them on a large range of mathematical modelling problems. At the end of the first decade of the current century, it befell to us to investigate the work-employment dynamics in Brazil by the implementation of a predator-prey model. The initial conjecture that established the investigation of a predator-prey symmetry in the work-employment system had, as one of their first motivations, the search for a deterministic model as opposed to the conventional statistical analysis, in our opinion, insufficient for a full understanding of the causal aspects of a sequence of events. It is not the case of a classical application of the standard Lotka-Volterra competition modelling, since the variables storage dissimilarities among population registers, not the populations themselves; this is so because the existing cyclic pattern which entangles the work-employment evolution is much more easily discerned by dissimilar modelling. Besides, while is tempting to discard the standard Lotka-Volterra model as too simplistic, there is a real situation where the detailed and complex dissimilar modelling holds undeniable utility. The aim of this article is to show that the work-employment system in Brazil admits a predator-prey modelling, providing managers with generalized theoretical elements that allow to a more accurate and deterministic understanding of the relations between employer and employee in Brazil. The model started from a simple correlation suggested by the superposition of two time series, one to the number of employed workers, and other to the number of active employers. The entanglement of the two series is analyzed

[1] Volterra did not cite Lotka because he didn't know his work at that time. Later, Volterra has no difficulty in recognizing the merits of his colleague, although he pointed out some differences: "I regret that I was not able to mention the interesting work of Dr. Lotka, which contains different applications of mathematics to chemical and biological questions." (free traduction from the authors).

with the aid of the clustering methodology. In the present article, the abstract analogy with a natural environment is the work market; the employer or corporation plays the rule of the predator; the prey is represented by the worker. The assumption of employers as predators has many reasons:

- The number of employers is much smaller than the number of workers (as in linx-hare, lion-zebra, crocodile-gnu and cheetah-baboon predator-prey relationships).
- Employers handle the number of workers depending on the profit margins which they want, heavily controlling the levels of unemployment.
- Employers often fail to collect the so-called "Guarantee Fund for Time in Service" related to their workers, causing irregularities that hinder the realization of labor rights in the event of contract termination agreement. Moreover, even without a contract being rescinded, the nonpayment of the Guarantee Fund blocks the employee to request it.
- Employers often fail to inform the CAGED (Brazilian abbreviation for General Cadastre of Employment)[2] on the movement of their staff, resulting in bureaucratic disarray that blocks the access to some labor rights in the event of loss of working papers; furthermore, the fail to inform the CAGED affects the labor statistics.
- Many employers absorb the workforce without formal contracts, depriving workers of their entitlements.
- However the scenario is changing in Brazil, many employers still do not make investments in quality, training employees and providing opportunities for better wages by productivity and capacity. The result is a high turnover, low wages and a working class that never achieves a status of participation on the employer's profits. In sum, we have a highly non-egalitarian society which does not prioritize education and manpower quality. Socially speaking, the predator is who foment social discrepancies and, in present case, they are employers (public or private). That is the point here.

From this point of view, we have not a model based on political preferences, as some watchers would think, but based on hard facts. There is no influence of maniqueist ideas, but simply findings. Paraphrasing Lawson in their studies (11), the offer of labor posts depends on the investment decisions of capitalists (employers), and the investment decisions of capitalists depend on the existing offer of manpower. Thus, since employers impose financial directions on the work market, often indifferent to the zeal for the rights of the working class, creating distortions and furling widespread dissatisfaction, it is reasonable to consider the adopted viewpoint in constructing the model. Obviously, in a more egalitarian society, firmly established on ethics and education, economically viable and with eyes in the future, it would be required a comprehensive review of the basic premises assumed. Also, in face of the facts itemized above, employers appear in many situations to dampen fluctuations in employee populations.

[2] CAGED is a great database updated monthly and containing all workers and employers in Brazil.

3 Observational premises

The data sources inquired were the CAGED and the RAIS (Brazilian abbreviation for Annual Report of Social Informations), two of the more important corporative databases of the Brazilian Ministry of Work and Employment. First of all we looked for harmonic patterns in the behavior of the working class and active employers. Figure 1 clearly presents the referred pattern in the chosen period (1996-2008). We defined active employer or company as one that updates CAGED every month (presenting or not variation in the number of admitted workers from a certain month to another). Secondly, we verify the presence of lags between the harmonic patterns of both series. This feature is also clearly visible in figure 1, with relaxation of the system all along the last fifty months. Table 1 shows a segment of the CAGED for the first year of the time series considered in present study.

Table 1: aggregate data of the CAGED database.

Period	Balance workers	Workers	Active employers
01/1996	-12.626	23.743.110	336.946
02/1996	-4.094	23.739.016	334.349
03/1996	10.003	23.749.019	359.248
04/1996	118.918	23.867.937	349.273
05/1996	172.930	24.040.867	344.896
06/1996	115.028	24.155.895	326.832
07/1996	68.920	24.224.815	338.098
08/1996	46.937	24.271.752	339.688
09/1996	88.964	24.360.716	327.525
10/1996	19.466	24.380.182	329.672
11/1996	-15.899	24.364.283	308.009
12/1996	-258.516	24.105.767	298.644

The original Lotka-Volterra predator-prey model makes several simplifying assumptions; it was adapted in many ways, as we may see in the vast literature on predator-prey interactions from which the authors selected some of the more important publications (see references). In many cases the former assumptions are relaxed or rearranged to fit some particular dynamics. It seems that the predator-prey model matchs, at least conceptually, the work-employment scenario, as it is very reasonable to admit that employers and workers are, after all, two populations conflicting with one another. Since the formulations of Karl Marx — with the analysis of the conflicts between worker and employer, and the entailments of such conflicts with structural elements formed by the connections among politic, social and economic plains — we understand that conflicts belong to the dialectic nature of the capitalist work relations. Also Richard Goodwin, in a different approach, found in Marx arguments conceptual resemblances that lead to his predator-prey dynamic model (12).

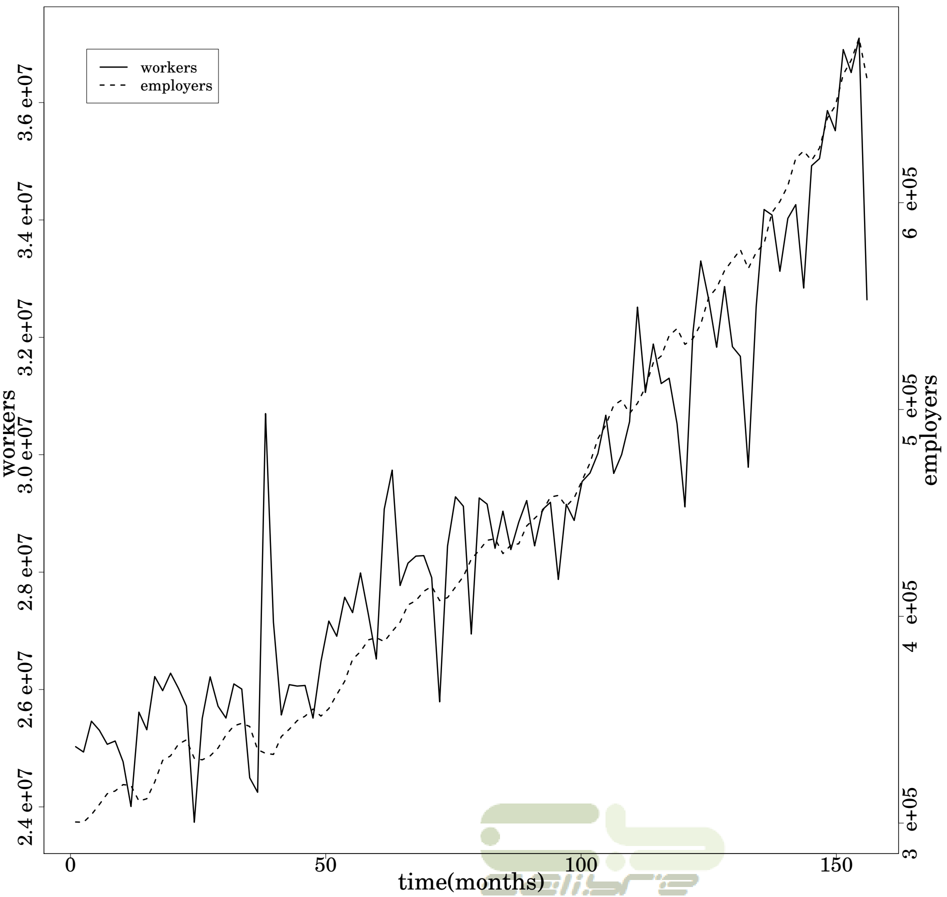

Figure 1: *harmonic patterns from workers (full line) and active employers (dashed line). The phase difference is clear in most of the peaks and valleys, especially in the fifty more recent months.*

4 Methods and simulations

4.1 Formal representation

As in the problem of Volterra, there exist circumstances inherent to the work-employment market (the environment of the theory) — for instance, those that depend on the changes of the global market caused by the influence of climatic fluctuations in the seasons, which lead to forced oscillations in production, or by politic crisis affecting one or more nations — that causes oscillations in the number of individuals at different profiles (employer and employee). Many of those circumstances are periodic, including politic ones, and clearly follow

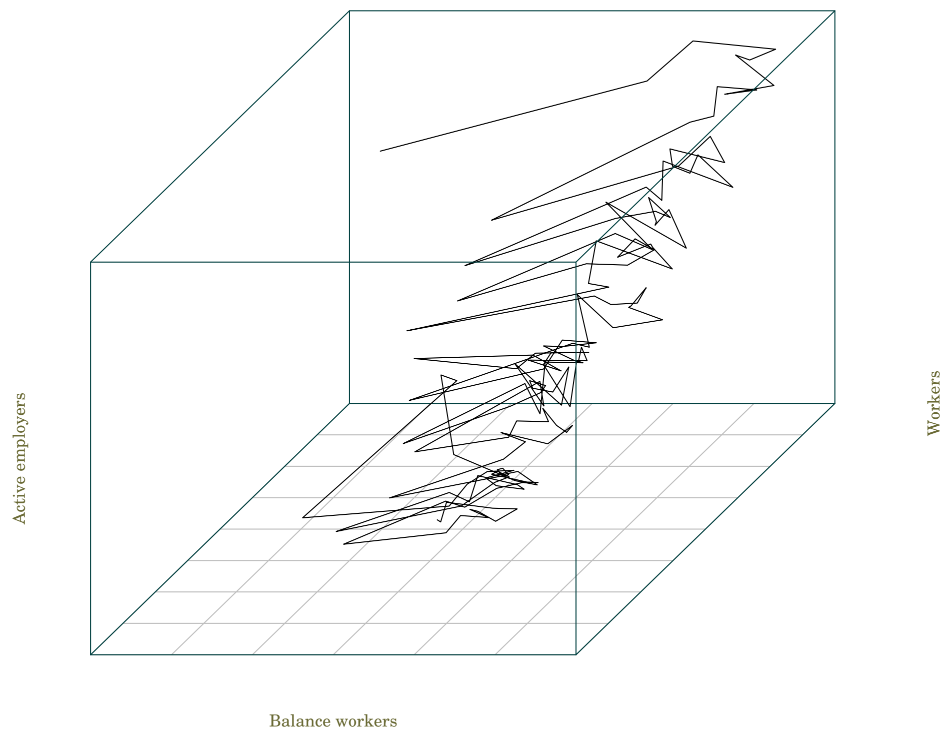

Figure 1-a: *the relationship of workers, active employers and balance of workers.*

rigorously deterministic representation, on the contrary of the mathematical models that engage tools of probability and statistics.

The pertinent information such a model can render is whether or not population abundances tend to an equilibrium at which both, workers and employers, will coexist and survive with few conflicts. The adopted model was based on the approach of Blasius and Stone (4) about oscillations with Uniform Phase Evolution and Chaotic Amplitudes (UPCA). An ecological UPCA model was chosen because it is suitable for treatment of two correlated periodic phenomena with chaotic amplitude varying in time, however, almost constant amplitude-independent frequencies as hinted in Figure 1. We applied the model to investigate the complex temporal phase synchronization in work-employment system at Brazil. In other words, our approach was based on the theory of synchronization of chaotic oscillations, defined in the most general case (8) as the entanglement between the phases of two coupled systems, while the amplitudes stay over chaotic regime in time. The real entanglement of phases in predator-prey relationships is much more complex than one could suppose at first. As pointed out by Holling, predator-prey interactions have shown that, in some cases of population densities, predators may control the number of preys, but this is not true to all density cases (5). However in Figure 1 populations grow and fluctuate in a net periodic way, the tests on the tangle between these populations goes far beyond the simplicity of that plot, as the reader can see in Figure 1-a. The balance of workers is a finite difference that indicates whether the work-employment scenario is favorable or not to the employee, and to what degree; it reflects the potential of the environment (work market) to sustain the system evolving, by analogy with the available natural resources in an ecological niche. In other words, if the balance is negative we believe that there is a retraction of work market to support occupied posts and, virtually, active employers. It is a variable that provides additional qualitative (positive or negative sign) information. Figure 1-a shows the active employers/workers-balance of workers relationship, illustrating the potential oscillations in these populations. The analysis of finite differences gives in general better information than punctual quantities taken one by one. A classic example from thermodynamics is the growth of entropy. Indeed, the point value of entropy would add little or no information, since entropy is a magnitude that increases monotonically until it reaches a transient value (equilibrium). Thus, what matters is the change in entropy between two transient states of a system. The Lotka-Volterra-Serpa differential equations consider a state vector formed by the product between a geometric tracing vector and a dissimilarity vector of observables. We use pairwise dissimilarities between registers instead of the quantities directly stored in data base just to compute the relative differences which are really cyclic all along the time series. That is the basic difference from the classical Lotka-Volterra approach. The dissimilarity matrix was made from the monthly populations of active employers and workers, and from the monthly balance of workers. This matrix computes all the pairwise dissimilarities (or "distances") between observations in the CAGED data set by means of the DAISY algorithm (13), that executes mixed measurements. We assume that the pairwise differences computed from the time registers determine the dynamical configuration of the system. Thereby, our model has three differential equations, one for the dissimilarity of balance of workers u (resources), one for the dissimilarity of workers v (preys) and one for the dissimilarity of active employers w (predators), such as

$$\begin{cases} \dot{u} = a.u.u_o - \alpha_1.f_1(u.u_o, v.v_o, k_1) \\ \dot{v} = -b.v.v_o + \alpha_1.f_1(u.u_o, v.v_o, k_1) - \\ -\alpha_2.f_2(v.v_o, w.w_o, k_2) \\ \dot{w} = -c.(w.w_o - w^{\dagger}) + \alpha_2.f_2(v.v_o, w.w_o, k_2), \end{cases} \tag{1}$$

where f_1 and f_2 represent either the Lotka-Volterra term $f_i(x, y) = xy$ or the Holling type II term $f_i(x, y) = xy/(1 + k_i x)$, and $w^{\dagger}$ denotes the minimum level of existent employers dissimilarity when there is scarcity of workpower or doldrums. The overdot indicates differentiation with respect to time. The quantities a, b and c are growth rates of balance of workers, workers, and employers respectively. The Holling type II functional response (5) introduces

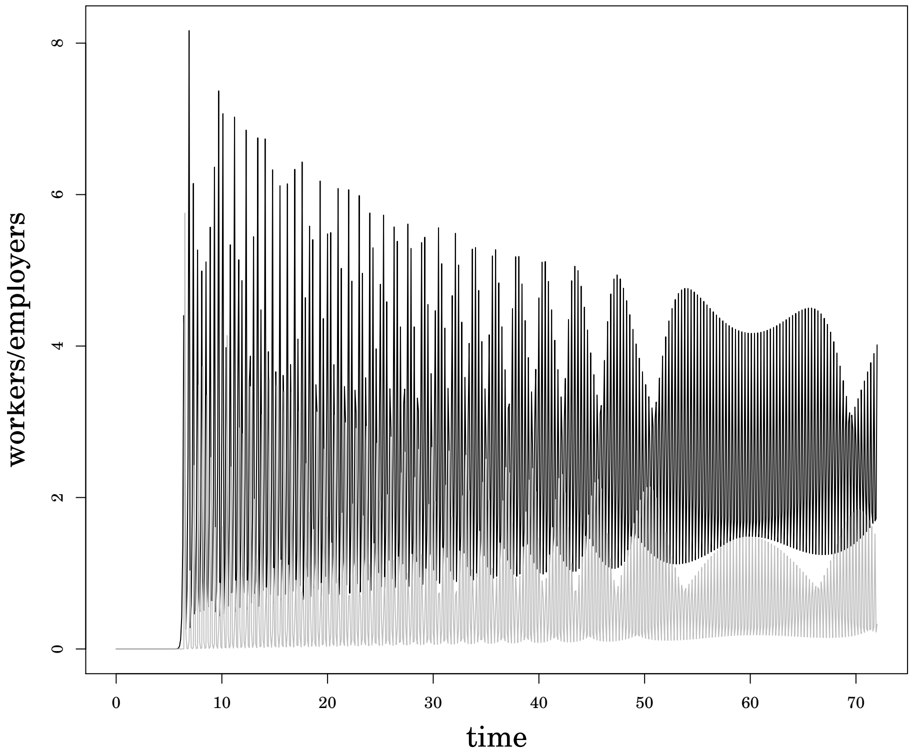

Figure 2: *UPCA oscillations of employer (grey) and worker (black) dissimilarities with initial time-delayed growth ($\alpha_2 = 1.0$).*

a decelerating intake rate related to the assumption that the dissimilarities of active employers is limited by its capacity to regulate dissimilarities of workers. Comparing to the original equations applied by Blasius,

$$\begin{cases} \dot{u} = a.u - \alpha_1.f_1(u,v,k_1) \\ \dot{v} = \ -b.v + \alpha_1.f_1(u,v,k_1) - \alpha_2.f_2(v,w,k_2) \\ \dot{w} = -\ c.(w - w^{\dagger}) + \alpha_2.f_2(v,w,k_2), \end{cases}$$

the reader can see that we merge variables u, v, z with u_0, v_0, z_0 and this changes the final geometric form of the system integration according to de observed data. Figure 5-b shows an example of UPCA oscillations. In our model, the positive constant k_2 measures the carrying capacity of employers to absorb manpower; the positive constant k_1 measures the carrying capacity of the work market to accept new workers. As pointed out by Kenneth Arrow and colleagues (7),

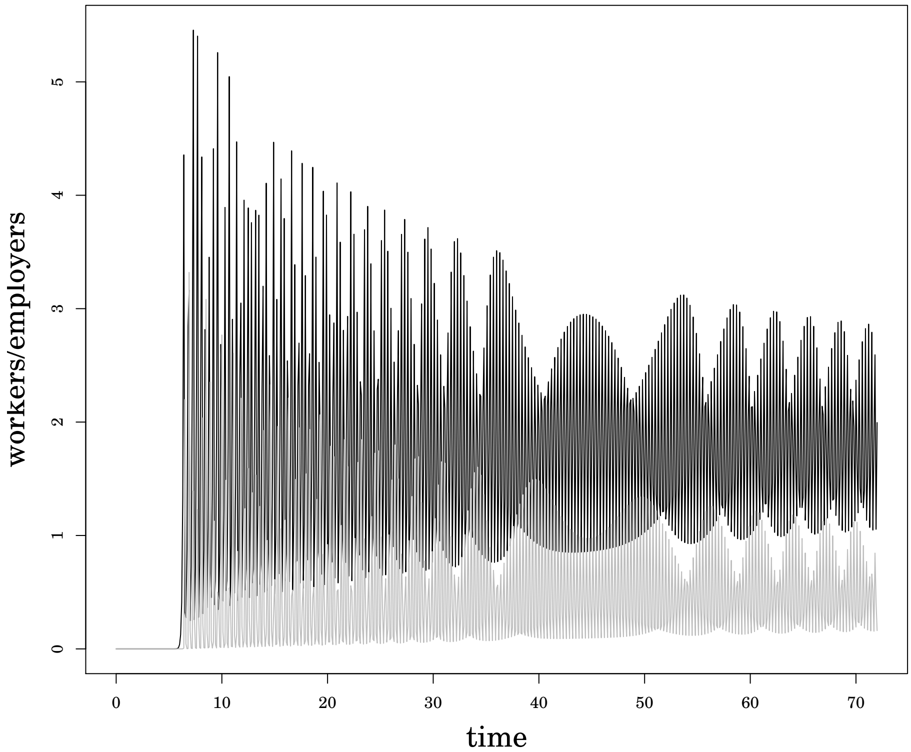

Figure 3: *UPCA oscillations of employer (grey) and worker (black) dissimilarities with initial time-delayed growth ($\alpha_2 = 1.4$).*

"*Carrying capacities in nature are not fixed, static, or simple relations. They are contingent on technology, preferences, and the structure of production and consumption. They are also contingent on the everchanging state of interactions between the physical and biotic environment. A single number for human carrying capacity would be meaningless because the consequences of both human innovation and biological evolution are inherently unknowable*".

So, it is interesting to investigate what would happen for work-employment scenarios with very low carrying capacity of employers to create new posts. For instance, if we take $k_2 = 0$ we means that the employers are saturated or temporarily disabled to open new posts, although k_1 remains greater than zero, that is, the work market remains in flux, but mainly due to high turn over. The contents of the differential variables were defined by means of the

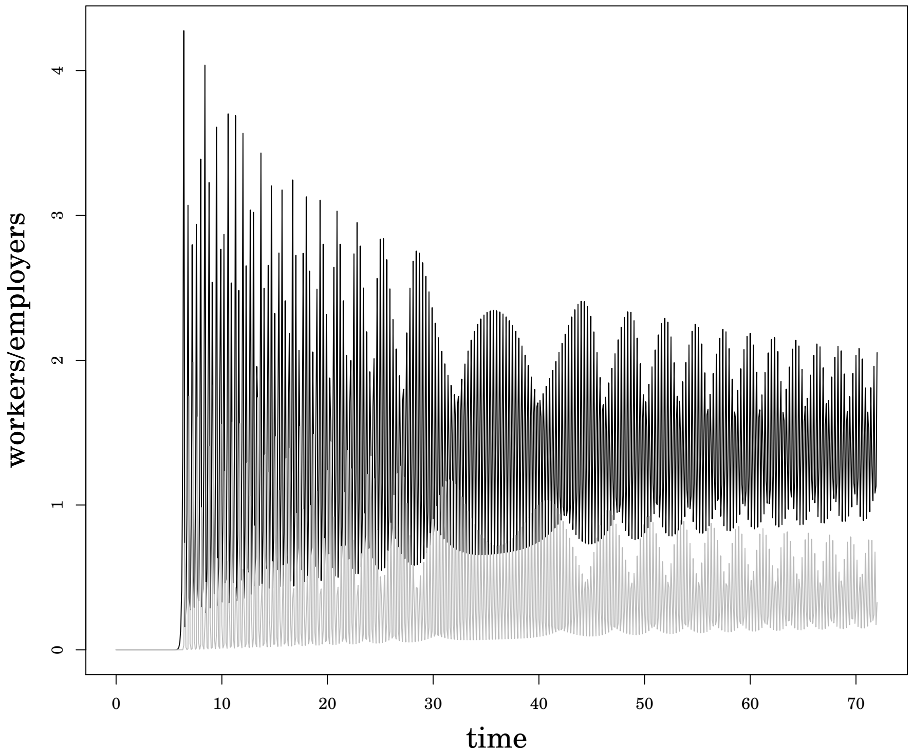

Figure 4: *UPCA oscillations of employer (grey) and worker (black) dissimilarities with initial time-delayed growth* ($\alpha_2 = 1.8$).

vectors,

$$
\begin{aligned}
&V\left(u := \omega[1], v := \omega[2], w := \omega[3]\right), \\
&U\left(u_o := \Pi[\delta,1], v_o := \Pi[\delta,2], w_o := \Pi[\delta,3]\right),
\end{aligned}
\tag{2}
$$

being $\omega[j]$ (j in $[1,2,3]$) the geometric components describing the state trajectories and $\Pi[\delta,j]$ the j dissimilarity components between observations with δ registers calculated according to the definition of Euclidian distance, that is,

$$
\Pi[\delta,j] \propto \left(\sum_{\kappa=1}^{d} |x_{\delta\kappa} - x_{j\kappa}|^r\right)^{1/r}, \tag{3}
$$

where d is the phase space dimensionality; $x_{\delta\kappa}$ and $x_{j\kappa}$ are, respectively, the $\kappa - th$ components of the $\delta - th$ and $j - th$ registers.

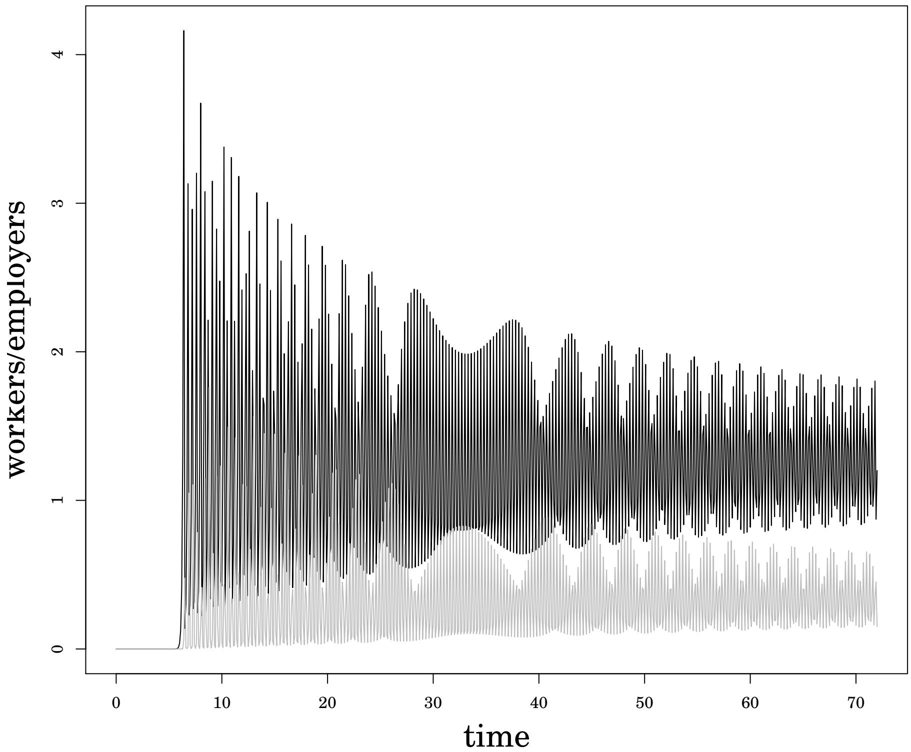

Figure 5: *UPCA oscillations of employer (grey) and worker (black) dissimilarities with initial time-delayed growth ($\alpha_2 = 2.0$).*

4.2 The equilibrium condition

No changes with respect to time in populations and resources, or in their dissimilarities, are represented by setting the equations equal to zero. So, the equilibrium condition for the system (1) requires,

$$\begin{cases} a.u.u_o - \alpha_1.f_1(u.u_o, v.v_o, k_1) = 0 \\ -b.v.v_o + \alpha_1.f_1(u.u_o, v.v_o, k_1) - \\ -\alpha_2.f_2(v.v_o, w.w_o, k_2) = 0 \\ -c.(w.w_o - w^{\dagger}) + \alpha_2.f_2(v.v_o, w.w_o, k_2) = 0 \end{cases} \tag{4}$$

Introducing the term of Lotka-Volterra (the simplest one), the equilibrium — where the dissimilarities of active employers, resources and workers do not vary — leads to,

$$\begin{cases} a.u.u_o - \alpha_1.u_o.v_o.u.v = 0 \\ -b.v.v_o + \alpha_1.u_o.v_o.u.v - \alpha_2.v_o.w_o.v.w = 0 \\ -c.(w.w_o - w^{\dagger}) + \alpha_2.v_o.w_o.v.w = 0 \end{cases} \tag{5}$$

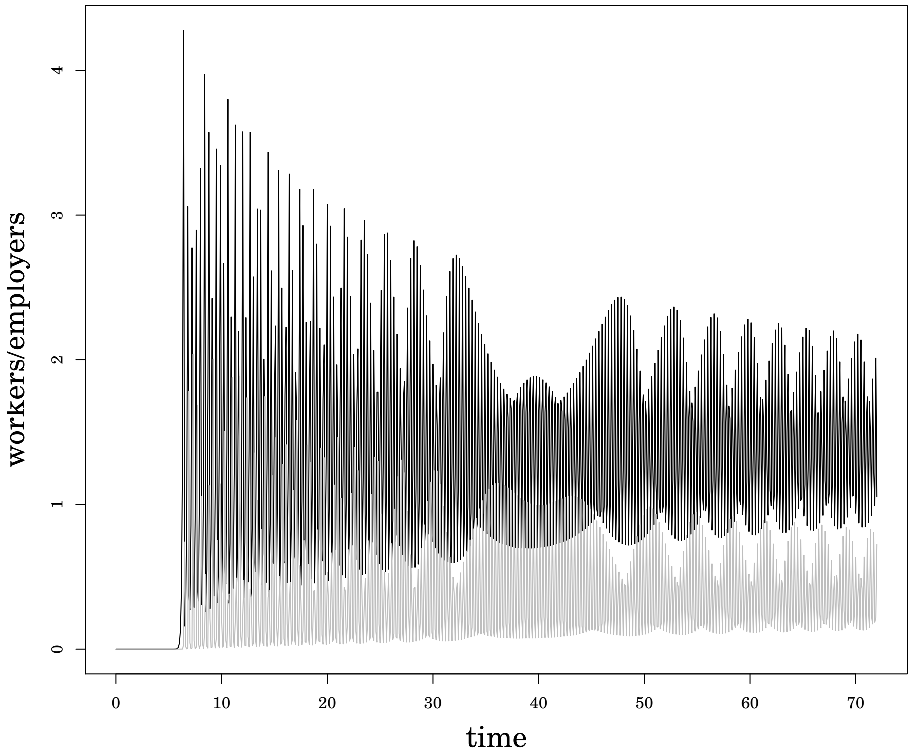

Figure 5-a: *UPCA oscillations of employer (grey) and worker (black) dissimilarities with initial time-delayed growth and* $k_2 = 0.00008$ $(\alpha_2 = 1.8)$.

From the first equation we obtain, whereas population dissimilarity shall be never equal to zero,

$$u.(a.u_o - \alpha_1.u_o.v_o.v) = 0, \tag{6}$$

$$a.u_o - \alpha_1.u_o.v_o.v = 0, \tag{7}$$

$$v = \frac{a.u_o}{\alpha_1.u_o.v_o} = \frac{a}{\alpha_1.v_o}. \tag{8}$$

Taking the third equation we get,

$$-c.w.w_o + c.w^\dagger + \alpha_2.v_o.w_o.v.w = 0, \tag{9}$$

$$w\left(-c.w_o + \alpha_2.v_o.w_o.v\right) = -c.w^\dagger, \tag{10}$$

$$w = \frac{-c.w^\dagger}{-\left(c.w_o - \alpha_2.v_o.w_o.v\right)} = \frac{c.w^\dagger}{c.w_o - \alpha_2.v_o.w_o.v}. \tag{11}$$

Finally, taking the second equation it follows,

$$\alpha_1.u_o.v_o.u.v = \alpha_2.v_o.w_o.v.w + b.v.v_o, \tag{12}$$

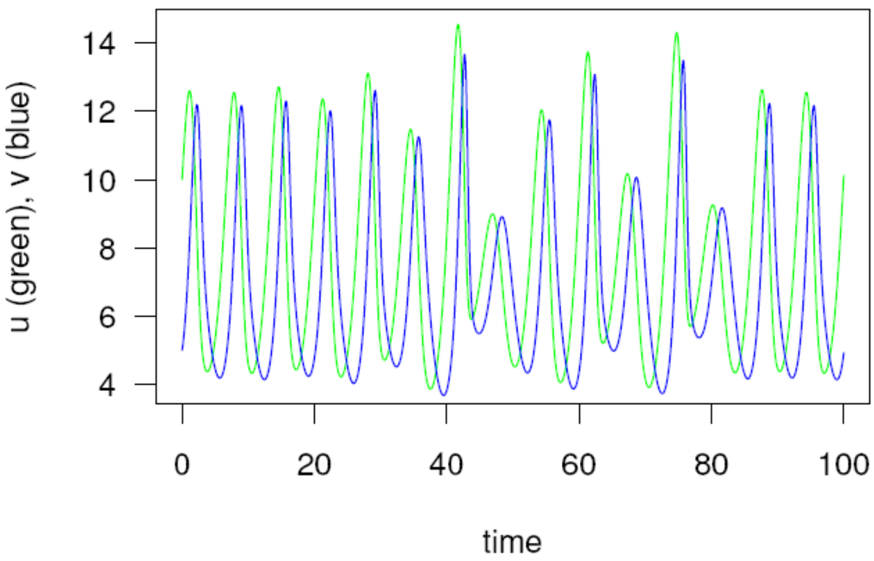

Figure 5-b: *simple UPCA oscillations by Blasius et al. for variables u and v.*

$$u = \frac{\alpha_2.v_o.w_o.v.w + b.v.v_o}{\alpha_1.u_o.v_o.v}, \tag{13}$$

$$u = \frac{\alpha_2.w_o.w + b}{\alpha_1.u_o}. \tag{14}$$

Substituting the value of w given by equality (11), it comes,

$$u = \left(\alpha_2.w_o.\frac{c.w^\dagger}{c.w_o - \alpha_2.v_o.w_o.v} + b\right)\frac{1}{\alpha_1.u_o}, \tag{15}$$

$$u = \left(\frac{\alpha_2.c.w^\dagger}{c - \alpha_2.v_o.v} + b\right)\frac{1}{\alpha_1.u_o}, \tag{16}$$

$$u = \frac{\alpha_2.c.w^\dagger + b.\left(c - \alpha_2.v_o.v\right)}{\alpha_1.u_o.\left(c - \alpha_2.v_o.v\right)}. \tag{17}$$

Now, applying the result set by expression (8), we have,

$$u = \left[\alpha_2.c.w^\dagger + b.\left(\frac{\alpha_1.c - \alpha_2.a}{\alpha_1}\right)\right]\frac{1}{u_o.\left(\alpha_1.c - \alpha_2.a\right)}, \tag{18}$$

$$u = \frac{\alpha_1.\alpha_2.c.w^\dagger + b.\left(\alpha_1.c - \alpha_2.a\right)}{\alpha_1.u_o.\left(\alpha_1.c - \alpha_2.a\right)}. \tag{19}$$

For the term of Holling type II, that is,

$$\begin{aligned} f_1 &= u.u_0.v.v_0/\left(1 + k_1.u.u_0\right), \\ f_2 &= v.v_0.w.w_0/\left(1 + k_2.v.v_0\right), \end{aligned} \tag{20}$$

the equilibrium sets,

$$\begin{cases} a.u.u_o - \alpha_1.u.u_0.v.v_0/\left(1 + k_1.u.u_0\right) = 0 \\ -b.v.v_o + \alpha_1.u.u_0.v.v_0/\left(1 + k_1.u.u_0\right) - \\ -\alpha_2.v.v_0.w.w_0/\left(1 + k_2.v.v_0\right) = 0 \\ -c.(w.w_o - w^\dagger) + \alpha_2.v.v_0.w.w_0/\left(1 + k_2.v.v_0\right) = 0 \end{cases} \tag{21}$$

The system becomes much more complex. Nevertheless, we fix $k_2 = 0$, meaning that the employers are temporarily disabled to open new posts, which determines the solution as it follows. The second equation leads to,

$$w = \frac{\alpha_1.u.u_0/\left(1 + k_1.u.u_0\right) - b}{\alpha_2.w_0}. \tag{22}$$

The first equation gives,

$$a = \alpha_1.v.v_0/\left(1 + k_1.u.u_0\right), \tag{23}$$

$$a.\left(1 + k_1.u.u_0\right) = \alpha_1.v.v_0, \tag{24}$$

$$1 + k_1.u.u_0 = \frac{\alpha_1.v.v_0}{a}, \tag{25}$$

$$u = \frac{\alpha_1.v.v_0 - a}{u_0.a.k_1}. \tag{26}$$

Finally, the third equation sets v as,

$$\alpha_2.v.v_0.w.w_0/\left(1 + k_2.v.v_0\right) = c.(w.w_o - w^\dagger), \tag{27}$$

$$v = \frac{c.(w.w_o - w^\dagger)}{\alpha_2.v_0.w.w_0}. \tag{28}$$

5 Discussion

Insecurity generated by the employers and the work market, over many years of restructuring and adjustment to globalization as the Nineties, led to an exacerbation of individualism. Competition became more dominant than cooperation. Fear and instability certainly promoted higher levels of depression, while millions of people reached the productive age, pressing more and more the formal work market. Moreover, due to the neo-liberal offensive (1980-1990) to the resumption of the "Theory of Human Capital", backed by Gary Becker, the wage differences came to be seen as responsibility of the worker himself, who thus became the chief culprit for all economic ills. The spreading of innovations due to technological changes (the convergence of information technology and telecommunications, and the continuing cost reductions) greatly affected the rate of hiring as a function of the resizing of the human effectives face to an accelerated automation. Also, Security in all the countries that care about

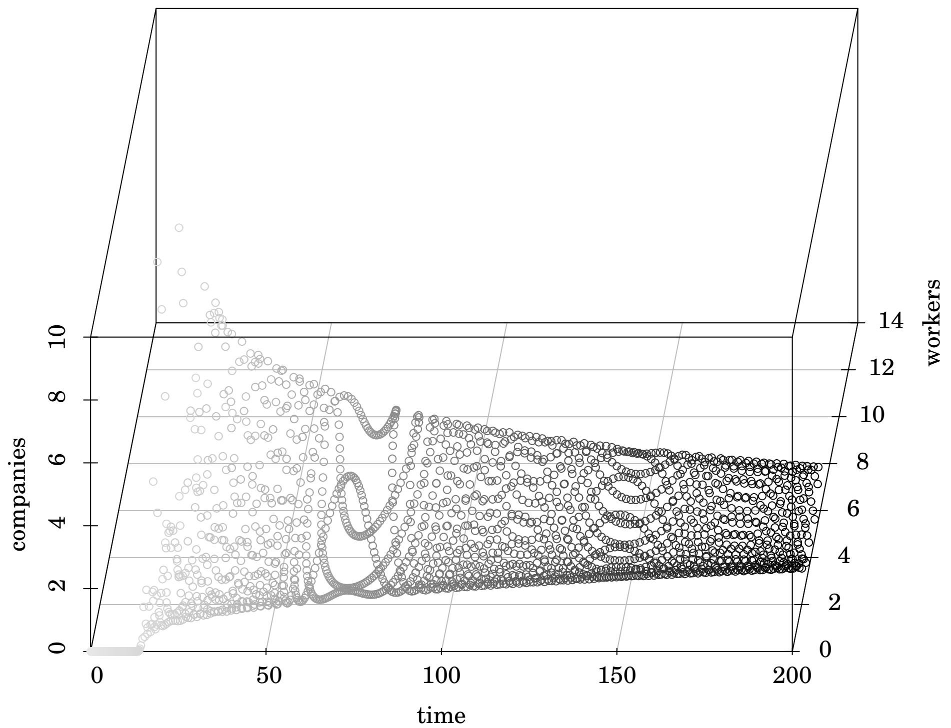

Figure 6: *the system in three-dimensional phase space.*

their retirees does not support the significant increase in life expectancy, discouraging retirements and thus retaining work posts occupied for much longer. All these factors contributed to a more chaotic work-employment scenario in Brazil from mid-1990 until about 2002 and they corroborate the interpretation of the plots. Many simulations are performed with vector V describing the geometry of the dynamics (theoretical), intrinsic to the nature of the system itself, and vector U storaging the measurable content (observational) of the dynamics. For the more realistic Holling type II functional response, with $k_2 = 0$, figures 2, 3, 4 and 5 show graphics of the general propensity of the system to less chaotic states accordingly the values 1.0, 1.4, 1.8 and 2.0 of the population interaction parameter α_2. Figure 5-a shows similar patterns for $k_2 = 0.00008$. Note that in all these figures there is a transition state to the phase of greater relaxation of the system. The great phase transition occurs close to the middle of the time series, from which the system goes on a relaxation stage. Comparing figures 1 and 5, the best fit realy seems to be for

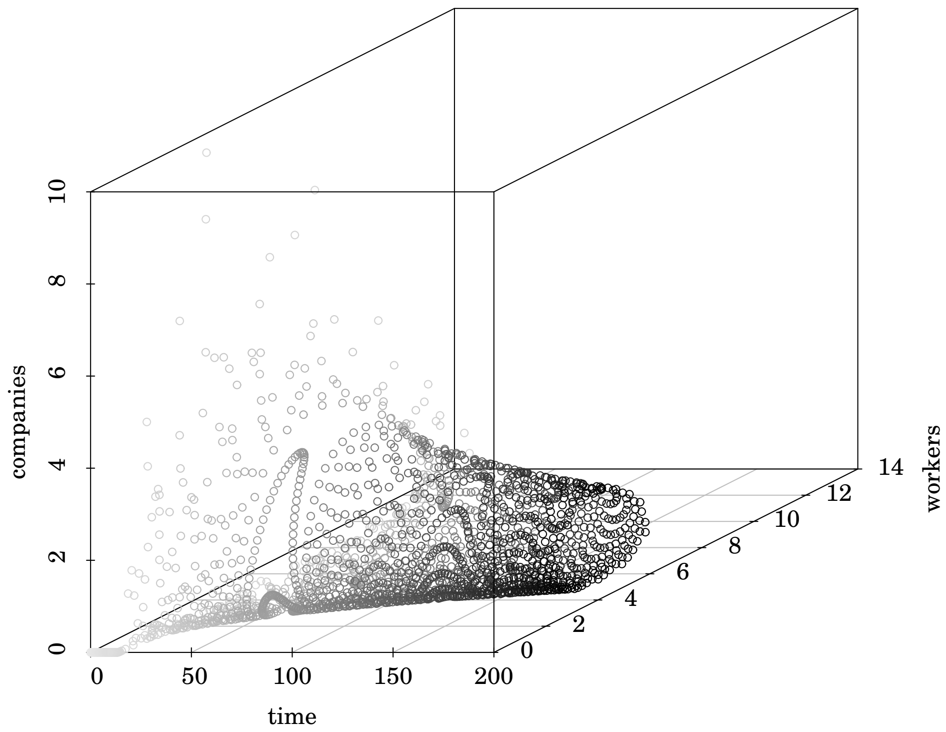

Figure 7: *another vision of the same system.*

$\alpha_2 = 2.0$ with phase transition around the zone between 50 and 100 months in figure 1. Figures 6 and 7 show the system's phase space for $\alpha_2 = 1.0$. The exact fitting of this parameter is a matter of more studies, since predator-prey stability could come from many factors, such as migration of manpower, employers self-limiting their staffs, seasonal recruitment due to foreseen peaks of sales, and so on, in the same way as pointed out by Andrzej Pekalski (6) in biological systems.

6 Conclusion

This study identified a predator-prey pattern in the work-employment system. The central equations applied are modified versions of those studied by Blasius *et al*, introducing pairwise dissimilarities between observations as a way to identify cyclic behaviors in the system. For carrying capacity $k_2 = 0$, simulations showed trajectories with initial chaotic behavior and tendency to final states of greater relaxation, which means that, even with zero carrying capacity of employers, the dynamics of the work-employment system in Brazil has demonstrated, during an interval of 12-15 years, propensity to adjusting to less chaotic regimes. In part, this is due to the great migration to solidary economy and autonomous work in the early years of the 20th century. Present article analyzed the system dynamics under the interpretation of the population interaction parameter α_2 as an index of the convergence between the aims of employers and workers or as an index of the equilibrium created by lower demand for formal employment. Of course, interactions and proliferation of workers and employers are not easily controlled, but the captation of manpower is a variable that may be monitored and partially governed by public policies. Moreover, as Grafton and Echenique (10) pointed out, we have the problem to choose the more appropriate ecological model to describe the situation in study. In our opinion, as more is learned about the system, we must change adaptively the model in order to lower uncertainty. Far from exhausting the subject, it is clear that in its general form the model represents reasonably well the dynamics of predator-prey for the work-employment system in Brazil. However, since our results stem from populist policies, and not permanent governmental social programs, the relative stability will become unsustainable. More recent data probably will show a very similar scenario to the early years of the time series. Therefore, further studies are necessary in order to test the model over the next few decades of the ongoing century.

7 Acknowledgements

The authors acknowledge Dr. Sergio Alves Cotia for their comments on the original manuscript and for his integral support to this work.

References

1. Lotka, A., Elements of physical biology. Williams and Wilkins, Baltimore(1925).
2. Volterra, V., Leçons sur la théorie mathématique de la lutte pour la vie. , Paris (1931).
3. Guerraggio, A. and Paoloni, G., Vito Volterra. Birkäuser, Springer Basel AG (2011).
4. Blasius, B. and Stone, L., Chaotic waves and phase synchronization in spatially extended ecological systems, in D. S. Broomhead, E. A. Luchinskaya, P. V. E.

McClintock and T. Mullin, eds., Stochaos: stochastic and chaotic dynamics in the lakes, American Institute of Physics, Woodbury, NY, 5ior02:221-225 (1999).

5. Holling, C. S., The components of predation as revealed by a study of small-mammal predation of the European pine sawfly, Canadian Entomologist 91: 293-320 (1959).
6. Pekalski, A., A short guide to predator-prey lattice models, Computer Simulations, Computing in Science & Engineering (2004).
7. Arrow, K. et al., Economic growth, carrying capacity, and the environment, Science, vol. 268 (1995).
8. Rosenblum, M. et al., Phys. Rev. Lett., 76, 1804 (1996).
9. Davis, H., Introduction to nonlinear differential and integral equations. United States Atomic Energy Commission, Washington-DC (1960).
10. Grafton, R. and Silva-Echenique, J., How to manage nature? strategies, predator-prey models, and chaos, Marine Resource Economics, vol. 12, 127-143 (1997).
11. Lawson, T., Reorienting economics: on heterodox economics, the mata and the use of mathematics in economics, Journal of Economic Methodology 11: 329-340 (2004).
    ··· Lawson, T., The (confused) state of equilibrium analysis in modern economics: an explanation, Journal of Post Keynesian Economics 27: 423-444 (2005).
    ··· Lawson, T., The nature of heterodox economics, Cambridge Journal of Economics 30: 483-505 (2006).
12. Goodwin, R., A growth cycle, in Feinstein, C.H. (ed.), Socialism, capitalism and economic growth, Cambridge University Press, Cambridge, (1967).
13. Kaufman, L. and Rousseeuw, P., Finding groups in data: an introduction to cluster analysis. Wiley, New York (1990).
14. Caughley, G. and Sinclair, A., Wildlife ecology and management. Blackwell Science Ltd, Melbourne (1994).
15. Earn, D. et al., A simple model for complex dynamical transitions in epidemics, Science 287, 667-670 (2000).
16. Spencer, P. and Collie, J., Patterns of population variability in marine fish stocks, Fisheries Oceanography, 6:188-204 (1997).
17. Tu, P. and Wilman, E., A generalized predator-prey model: uncertainty and management, Journal of Environmental Economics and Management 23, 123-138 (1992).