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Type-Ia Supernovae, Clusters of Galaxies and a New Inquiry on the Inhomogeneous LT Cosmology

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Abstract This article discusses some features of modern astrophysical cosmology in light of what "is not known" about type-Ia supernovae, clusters of galaxies and the evolution of the universe. The study summarizes a work begun in 2007 plus observational updates over the last 10/12 years, and disruptive personal theoretical propositions. On the trail of a critical thinking, it seeks to point out pivotal questions to which we will only find answers from a broad and careful review of the premises currently accepted at the heart of cosmology, today more observational than ever, and still pending a theoretical structure globally consistent with the universe observed on large scale.

Key words: Astrophysical cosmology, type-Ia supernovae, clusters of galaxies, structures, sub-Plankian domain, general relativity.

Resumo. Este artigo discute alguns aspectos da moderna cosmologia astrofísica à luz do que "não se sabe" sobre supernovas tipo-Ia, aglomerados de galáxias, e sobre a evolução do universo. O estudo resume um trabalho iniciado em 2007 acrescido das atualizações observacionais dos últimos 10/12 anos, e de proposituras teóricas pessoais disruptivas. Na trilha de um pensamento crítico, procura apontar questões basais para as quais somente encontraremos respostas a partir de uma ampla e cuidadosa revisão das premissas atualmente aceitas no cerne da cosmologia, hoje mais observacional do que nunca, e ainda pendente de uma estrutura teorética globalmente consistente com o universo observado em larga escala.

Palavras-chave: Cosmologia astrofísica, supernovas tipo-Ia, aglomerados de galáxias, estruturas, domínio sub-Plankiano, relatividade geral.

Introduction

My interest in type-Ia supernovae started with the research I conducted on inhomogeneous models of the universe — exact solutions of Einstein's equations in which quantities such as curvature and expansion field vary point-to-point in space-time —, during the development of my master's dissertation, for which several works provided important subsidies ([6], [7], [8], [9] and [10]). The general context of this interest, as it could not be otherwise, was based on the classical cosmology of Friedmann, Lemaître, Robertson, Walker (FLRW), from which inhomogeneity was introduced.

The FLRW cosmology arouse by the assumption of a homogeneous and isotropic universe at large scale and, ultimately, is characterized by two functions: an average expansion rate of the universe (H) and the density parameter (ω), both of which depending on time. Their values, although not observed directly, are deduced as of information extracted from the light that reaches us since the past through the "cone of light". There is, as in all physical science, a subjective element of interpretation in the construction of this information, which met the evidence well until the arrival of a copious collection of data obtained from type-Ia supernovae, the distribution of galaxies and the anisotropies found in cosmic radiation background. It is in the scenario of discrepancies arising from the new discoveries that "dark energy" emerged as the redeemer of order. Through the reintroduction of a cosmological constant, until now without any theoretical model that explains its origin and magnitude, it warrants an accelerated expansion of the universe and a "justification" for the apparent dimming of luminosity seen in distant type-Ia supernovae.

Til then, all theoretical and interpretative efforts characterized an obstinacy to preserve a homogeneous description, in my opinion quite controversial, and has given rise to a range of works aimed at increasingly affirming the inhomogeneous image that technological modernity, in obtaining precise data, has allowed us to glimpse, especially with regard to the study of type-Ia supernovae. At the time I started the first work, my main task was to describe an inhomogeneous Lemaître-Tolman (LT) universe viable with the same level of detail achieved in the description of its homogeneous alter ego FLRW, highlighting the definition of a gravitational refractive index in LT cosmology due to a weak lensing effect.

Regarding type-Ia supernovae (hereinafter SNIa), over the early years of the current century, it was prevailed the theoretical representation of a thermonuclear explosion of a carbon/oxygen white dwarf star in the process of dredging up material from a companion, usually a red giant (for some time now, a binary model of two white dwarfs interacting in a deadly dance has been considered). The mass of the dwarf star tends to the Chandrasekhar limit, at the same time that its temperature and density converge to the melting point of carbon. The energy released during this process ends up exceeding the cooling rate due to the expansion and loss of neutrinos, so that the star is unable to maintain its integrity, exploding violently. This scenario is independent of cosmological parameters — which means that such events could have occurred at any time —, leading us to believe that SNIa would be standard candles, from which it would be possible to infer distances on cosmological scales.

In summary, this article is dedicated to an analysis of cosmology in an inhomogeneous scenario. The proposed models do not assume perturbative methods. Some aspects inherent to the nature of the universe which are independent of solutions to Einstein's equations are discussed. Much of the basic theory explained was inspired on results transmitted by Garfinkle [6] and Enqvist [10] (always taking into account the examples of the three typical functions of LT modeling given by the first), forming the background of my recent contributions. With respect to SNIa and galaxy cluster data, the most current compiled surveys available at the time of preparing the graphs presented were assumed.

The essentials of SNIa

SNIa are characterized by no hydrogen in their spectra and strong lines of Si, Ca and Fe. They are among the largest thermonuclear explosions in the universe. The light from such explosions is capable of traveling very long distances. Although they led to the discovery of the acceleration of the expansion rate of the universe, a large number of uncertainties remain in current theoretical models. With no doubt, calibration and size of SNIa samples at low redshifts have been substantially enhanced during the first two decades of the 21st century. Furthermore, computational modeling has offered excellent perspectives, mainly with regard to the distribution of these phenomena, despite the challenging task of dealing with the immensity of distances and time scales. However, we do not know of a robust theoretical model capable of reproducing the power observed in explosions, which are even brighter than the host galaxies. Also, the simulations only account for events with very low luminosity compared to the mean luminosity deduced from the available sample. Lastly, although the most accepted model of the genesis of these events considers a binary progenitor system, there is no consensus on which star would be the donor in the matter accretion process. Because of this gap in knowledge regarding the progenitor system, it is to be expected the appearance of light curves (magnitude as a function of time) that are not as similar to each other as we would like, hence the need to "standardize" these curves. Taking into account the errors and factors that contribute to the dispersion of the light curves of different SNIa, it is possible to adjust them so that they indicate the same absolute magnitude, i. e., the same total emitted power, making it viable to use them as standard candles. Therefore, the more of them are observed, the better the accuracy of the luminosity estimate.

In recent times, an intense process of investigation began with the aim of verifying whether realistic inhomogeneous models without a cosmological constant could account for the weakening of the light emitted by SNIa, in such a way as to interpret it as an epiphenomenon capable of mimicking the acceleration of the universe. It is known that LT models can be fit to a wide set of observational data, which means it is possible to fit them to SNIa data. However, if the model does not fit other sources, the cosmological problem will remain unsolved. The set of observational data has modified the state of cosmological knowledge insomuch that, considering the three main sources of data currently recognized, we have the following values for the average density of matter:

- Cosmic radiation background $\rightarrow \Omega_M \sim 1$,
- Surveys of galaxies $\rightarrow \Omega_M \sim 0,3$,
- Type-Ia supernovae $\rightarrow \Omega_M = 0,28 \pm 0,10$.

To banish similar discrepancies, it was agreed to introduce a cosmological constant Λ, or vacuum energy Ω_Λ, into Einstein's equations. This artifice leads to an accelerated expansion of the universe. Consequently, the apparent weakening of SNIa finds its natural explanation in a homogeneous universe. Although this ensures excellent agreement of the ΛCDM model with the observational data, the procedure brings with it some disturbing aspects. Viewed from the perspective of Planckian units, the cosmological constant appears as an absurdly small quantity (smaller by around 120 orders of magnitude). Several theories have been used to clarify the reasons for this value. The role of supersymmetry, for example, in evaluating the cosmological constant requires the incorporation of gravity into its theoretical framework, something that is done by introducing supergravity. With it, ordinary particles are complemented by a gravitational multiplet comprised by the graviton and its supersymmetric partner, the gravitino. The graviton, with spin 2, is the mediator of gravity just as the photon is the mediator of electromagnetism. Gravitino is a spin 3/2 particle. If there is no supersymmetry breaking, both particles remain massless. In the simplest supersymmetry-breaking scheme, the gravitino becomes massive. As soon as supersymmetry is broken, the vacuum energy assumes values of the order of 10^{40}Gev, far above what would be expected for a plausible physical driver of the cosmological constant as currently estimated. These conclusions, however, are based on the subsumption of two particles whose existence has never been proven, and perhaps a broad review of the accepted model is necessary here (for a complementar discussion on related subjects, see references [2], [11], [12], [15], [17] and [18]). Furthermore, the ΛCDM model requires that we live in a *sui generis* cosmological era in which matter and dark energy have comparable densities. Finally, all the effort spent over decades by particle physicists to show that the cosmological constant must be zero will fall apart if we definitively accept it with its astonishingly tiny, but not zero, value in the bases of cosmological knowledge. It was above all these disturbing facts that motivated the search for alternative models to the standard model. However, the astonishing smallness of some physical magnitudes may not be as strange as it seems. I will briefly review what has been done in terms of SNIa, comparing the accepted findings with my recent propositions.

One of the streams of research consists of assuming a LT model as simple as possible, verifying the impact of the inhomogeneous distribution of matter and the non-uniform expansion of the universe on the propagation of light [2]. Apparently, the first analyzes showed that on small scales variations in density actually induce variations in brightness; the same does not occur on large scales. Brightness fluctuations decrease with distance, a fact that makes an explanation without cosmological constant unfeasible once very high redshifts are considered. Due to the great uncertainties involved and the relative scarcity of SNIa, the subject is still a source of controversy.

Astronomical observations of the "local universe" indicate that its density varies from low values related to voids to high values related to agglomerations. Table 2 displays five models that illustrate this fact. Measurements on the distribution of matter imply that the density contrast ($\delta = \rho/\rho_b - 1, \rho_b =$ density of the background) varies by $\delta \approx -1$ in voids up to δ equal to several tens in clusters. Such structures exist in diameters from several Mpc to tens of Mpc. However, if the average is considered on large scales, the density remains between $0.3\rho_b$ and $4.4\rho_b$, with the sizes of the structures being on the order of several tens of Mpc. To date, there is no observational evidence of structures larger than superclusters, that is, with diameters on the order of hundreds of Mpc or more.

Table 2: Form of the functions used for different LT density models (ρ_b = background density).

Model	Function
1	$\beta = 0; \rho/\rho_b = 0,5 + 0,2\cos(10^{-5}\pi r Mpc^{-1}) + 0,5\cos^2(10^{-5}\pi r Mpc^{-1})$
2	$\beta = 0; \rho/\rho_b = 0,4 + 0,6\cos(2x10^{-5}\pi r Mpc^{-1}) + 1,8\cos^2(2x10^{-5}\pi r Mpc^{-1})$
3	$\beta = 0; \rho/\rho_b = 1 + 0,4\cos(10^{-5}\pi r Mpc^{-1})$
4	$\beta = 0; \rho/\rho_b = 1 + (8x10^{-6} r Mpc^{-1})^{0,55}$
5	$\rho/\rho_b = 1$

The models presented in table 2 provide preliminary estimates of density behavior. The symbol β represents the time function of the Big-Bang. Note that the assumption of $\beta = 0$ was made for the early stages of the universe and arises from observations of the cosmic microwave background. Such observations imply that the universe was quite homogeneous until the moment of the great final dispersion of the Big-Bang. Therefore, the amplitude of β could not be greater than a few thousand years, which compared to the current age of the universe is negligible. In summary, if β assumed a high value under initial conditions, temperature fluctuations would be greater than those observed in the cosmic microwave background.

In model 1, most of the regions through which supernova light propagates are of low density. In model 2, the densities found are, for the most part, higher than the background density. In model 3, the average density is on the order of the background density itself. In particular, I will only focus on models 4 and 5, which deserve more attention. In model 4, the initial value of the Big-Bang time function, $\beta = 0$, is consistent with the cosmic microwave background. The density distribution, monotonically increasing from a mean value $\rho = \rho_b$ at the origin to $\rho = 2.5\rho_b$ at about 3 Gpc, was chosen in order to fit the supernova observations. An increase in density leads to a decrease in expansion. However, even though it is expected to obtain good approximations from much more accurate surveys, there are no systematic observations of the density distribution, or even expansion, at distances on the Gpc scale. All we know boils down to the fact that the Milky Way's motion with respect to the cosmic microwave background is small. Therefore, to account for the relatively small movement in contrast to the cosmic background radiation, taken as a reference at rest, we have to admit the increase in the expansion of the universe at large distances. It is worth highlighting once again that the great flexibility of the LT models allows the choice of functions that satisfy the adjustment to data originating from the cosmic background radiation. In model 5, the density distribution is assumed to be equal to the background value, $\rho = \rho_b$. This indicates that there are no structures on the Gpc scale. The Big-Bang time function is quite inhomogeneous, making it strongly inconsistent with observations of the cosmic microwave background.

A universe that folds in on itself: The reentrant galaxies model

Once upon a time, the idea of a Big-Bang sounded naturally human, a kind of scientific creationism, the anthropoethical intuition that everything has a beginning and, probably, an end. This may be so, but I think the conjecture of a universe that repaginates itself throughout eternity is inevitable. I believe it is sensible, given the most current information, to speak not of an absolute origin but of the beginning of an evolutionary stage that culminated in the first galaxies and that allowed the emergence of life as we know it. The dynamics of the universe is complex and may even be unintuitive.

It's worth a reflection here. Disruptive thinking in physics has never been more necessary than now [23], after the reiteration, through the James Webb telescope, of the existence of mature galaxies where they should not be, according to the standard model. At this point, the term "Big-Bang time" seems a bit embarrassing, so perhaps it is more prudent to speak of a conventional "ground zero time", much earlier than current model predicts, designating the theoretical beginning of a certain peculiar phase of the universe that we do not yet understand well.

Taking a closer look at the situation, the so-called 'impossibly early galaxy' problem became more serious after recent confirmation of many massive galaxies in twilight of their lives observed at $z \approx 3$ (see the massive quiescent galaxy ZF-UDS-7329 with photometric redshift > 3 [27]) when the universe was only 2 Gyr old. This unexpected and disconcerting fact contrasts to the prediction consensus of the hierarchical ΛCDM model, in which galaxy assembling, a long term gradual merging of several progenitors, took place at $z \approx 0.7$, when the universe was 7 Gyr old. Given such inconsistencies, I have been working on a hypothesis, which I called the "reentrant galaxies" model. This is a topological approach.

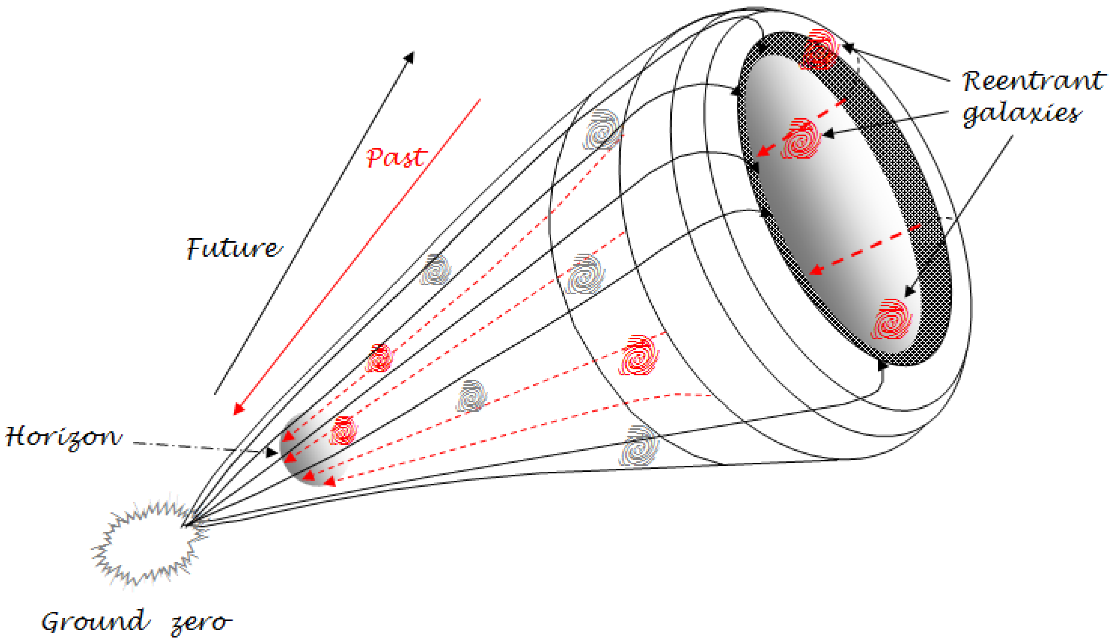

Figure 1: *Schematic model of reentrant galaxies. Note that the time-bulb expands into the future with the galaxies in black emerging from a time after the horizon of approach of the galaxies in red, towards the past. For this reason, we see mature galaxies where they could not be if we considered only the most intuitive evolutionary part of the universe.*

Intuitively, we think of the simplest forms of expansion, imagining something like an almost perfect spherical symmetry, for example. But there are other possibilities. Suppose that the cosmic woof expands within a time-bulb (Figure 1), an imaginary cosmic conduit along which from a given moment the four-dimensional sphere begins to fold inwards on itself[1], as if we were pressing a rubber balloon with a finger (this is not a rupture, but rather a depression in space-time woof). Then, we get a sphere with a "hilo". We can recall Poincaré's conjecture, whereby if a three-dimensional manifold has no topological "holes" and all its parts are continuously connected, then it must be a three-dimensional sphere (any closed arc on the manifold surface can be continuously reduced to a point). Similarly, our *hilosphere* is homeomorphic to a four-dimensional sphere. So, the first oldest galaxies would be dragged by the hilo — the reentrant warp — "toward the past", diving into the depression asymptotically until a horizon that inexorably separates us from the ground zero (even without thinking about a cosmological constant, we may consider decreasing rate of expansion towards the horizon). Note that the theoretical time-bulb expands in all directions, including the reentrant direction — the hilo — which leads to exotic compositions between redshifts (between a normal and a reentrant galaxy, there is the redshift referring to the deviation due to the global expansion of the bulb, and the redshift referring to the deviation due to the reentrant expansion, i. e., the inner expansion of the hilo). It seems to me that it is a model capable of shedding light on several issues, but at the cost of some review in the essence of the standard model. By homeomorphism, we can map all galaxies spherically, more or less as if we were constructing a *galasphere*. Obviously, someone will ask why there would be a sphere with a hilo! The answer is very simple: we don't know, this is only a model. It's not very different from asking why the Big-Bang. The fact is that new records of old galaxies will probably populate our compiled surveys, perhaps not many. In any case, the hierarchical model of galaxy formation is currently in check.

The above results and conjectures suggest that the only way to fit supernova data is to assume the decreasing expansion of the universe toward the past. This can be done either by considering a decrease in acceleration with the radial coordinate (models 4 and 5) or by adopting a cosmological constant (standard approach). The first hypothesis does not require the introduction of the cosmological constant, however, it requires that we are located in a privileged

[1] The conduit represents the historical sequence of expansion states of the quadrisphere.

position within the universe, as well as the existence of structures on the Gpc scale. The second hypothesis admits that the models suggested here support the acceleration of the universe as a justification for the observations made on SNIa. Within the framework of such models it is impossible to fit supernova data with a realistic distribution of matter, where variations in the contrast of density were similar to those observed locally. Apparently, the two alternatives are equivalent from the point of view of observational analysis. The difference lies rather in philosophical assumptions. There's no way to decide if we really are in a special place. Due to the flexibility of LT cosmology, models 4 and 5 can be fitted to the cosmic microwave background data by simply assuming that the structure in Gpc is compensated by external regions. The reader should note that for such models a distribution of matter in spherical symmetry was assumed, with the exception that this assumption is suitable for the propagation of light in a short time interval. For longer periods, the evolution of matter becomes important. It is generally accepted that the universe evolves very little up to redshifts of approximately 0.5, so the analysis mentioned here does not differ significantly from reality.

Dark energy as expansion energy

Do we fear a revision of the standard model? If so, it's understandable. After all, many scientific careers were based on ideas, representations and interpretations anchored in the concept of the Big-Bang and in the belief of a universe with a primitive inflationary stage, later evolving in a homogeneous and isotropic condition. If someone, after so many decades, raises the suspicion that perhaps we are misinterpreting the redshift, this will not be easy to digest. Resistance to change is normal, as long as it does not fall into the sad situation described by Arp in 1987:

"[...] *recientemente se han producido, por parte de determinadas personas, intentos de hacer desaparecer nuevos resultados que no concuerdan con sus particulares puntos de vista*" [1].

Science is not immune to the flaws of human character. For me, I like to think of a science that is open, detached and ready to give way to new suspicions.

So, if there is no disruptive thinking in physics, we will remain embarrassed for a long time in fundamental questions. My endeavor to understand dark energy has led to imagine it as the energy of the sub-Planckian expansion of space-time itself, which makes us deal with uncomfortably small things. It is not easy to overcome the taboo of cosmic censorship because we know the limitations of instrumental sensibility, but if we assume a four-dimensional space-time continuum it would be absurd to limit it to our finite observation capacity. According to the sub-Planckian expansion model of the continuum that I have been developing for some years, the discrepancy in magnitude compared to the theoretical LT curve certainly includes the locally inhomogeneity related to the interactions between dark energy and the energy released by SNIa, which points to an inhomogeneous expansion in the vicinity of the event. However, it will be difficult to separate the part of expansion corresponding to this interaction until both the explosion model and dark energy are better understood.

The hypothesis of a homogeneous universe can be interesting, and certainly simplifies the theoretical foundation and brings promising results, but there is an ontological problem in the intimate relationship between man and the universe: we are inside the universe and cannot encompass it completely. Furthermore, inhomogeneities occur in everything we reach with our instruments. A persuasive example is the so-called *KBC* void, referring to the large vicinity of the Milky Way, whose average density of matter is notably lower than the average in the observable universe. The photometric characteristics of SNIa themselves seem to depend on the environmental conditions of their birthplaces, whether their host galaxies are found in clusters or in open fields. Also, as I reported in previous work [24], an accurate work by Migkas and colleagues [20] discusses the common assumption of isotropy of the late Universe, testing the anisotropy on the X-ray galaxy cluster scaling relations (Figure 2) with important outcomes following many studies reporting deviations from isotropy under different cosmological probes. It is also evident that life is a rare phenomenon, so that biological activities constitute a largely inhomogeneous scenario. So, a homogeneous universe is defended, but it is the inhomogeneities that bring us knowledge about the most impressive nuances of its evolution.

Warping geodesic intervals

I think of dark energy as forming a faint uniform distribution exotically exerting negative pressure, manifesting from sub-Planckian levels of space-time. Dark energy as expansion energy was introduced from the postulate that a sub-Planckian interval of space-time expands spontaneously as a natural characteristic of its own,

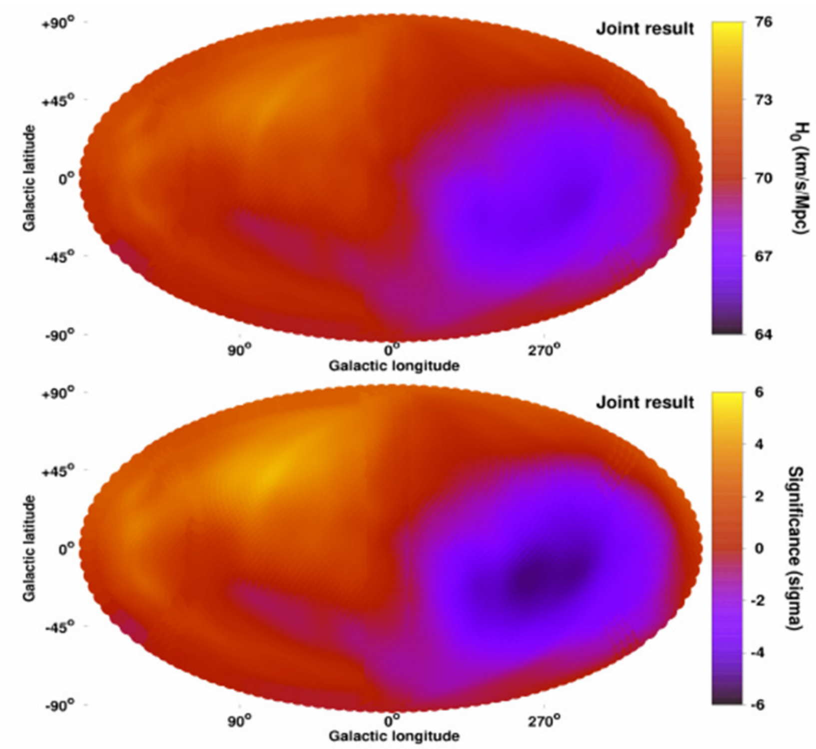

Figure 2: *H_0 anisotropy map on the X-ray galaxy cluster scaling relations (courtesy of K. Migkas, Astronomy & Astrophysics, 2021).*

and is blind to cosmology. In other words, the intrinsic behavior of space-time is independent of solutions to Einstein's equation. The role played by that dark energy can be well understood from the expression of the Euler-Lagrange modified sine-Gordon type equation of the geodesic line referring to the static metric $ds^2 = -e^{2\phi(\mathrm{X}^1,\mathrm{X}^2,\mathrm{X}^3)}dt^2 + \zeta_{ij}(\mathrm{X}^1,\mathrm{X}^2,\mathrm{X}^3)d\mathrm{X}^i d\mathrm{X}^j$, in which an arbitrary sub-Planckian interval subject to expansion or contraction is warped by a soliton [21], say

$$\frac{d}{ds}\left(\zeta_{ij}\,\widetilde{\mathrm{X}}^j\right) + \frac{\partial\phi}{\partial \mathrm{X}^i}e^{2\phi}\tilde{t}^2 - \frac{\partial\zeta_{jk}}{\partial \mathrm{X}^i}\frac{\widetilde{\mathrm{X}}^j\,\widetilde{\mathrm{X}}^k}{2} + \\ + m^2\tilde{t}E\sin\vartheta\frac{\partial\vartheta}{\partial \mathrm{X}^i} = 0, \tag{1}$$

corresponding to the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\left(-e^{2\phi}\tilde{t}^2 + \zeta_{ij}\,\widetilde{\mathrm{X}}^i\,\widetilde{\mathrm{X}}^j\right) - m^2\tilde{t}E\,(1-\cos\vartheta) \tag{2}$$

for

$$\frac{d}{ds}\left(\frac{\partial\mathcal{L}}{\partial\,\widetilde{\mathrm{X}}^i}\right) - \frac{\partial\mathcal{L}}{\partial \mathrm{X}^i} = 0. \tag{3}$$

For the sake of physical meaning and not of pure mathematical manipulation, the soliton in the theory presented is a pulse in the very woof of space-time, an eventually spontaneous excitation, and not an inference from a solitonic solution of Einstein's equation. It was introduced as an external physical motivation acting on the natural and spontaneous expansion. As I pointed out early, "the arbitrary constant E matches the freedom of the null geodesics affine parameter and is interpreted as the expansion energy contained in the worldline intervals". Analyzed in such small intervals, the energy inherent to the natural expansion of space-time will be unimaginably small [21].

The structure of a time-like peace of a geodesic is typified by the continuously expanding interval $\mathrm{X}^0 = \langle\forall|\, \tau - \tau_0\rangle$ (read "no matter the scale of $\tau - \tau_0$") as a finite time-like path (always expanding, no matter what scale we consider it) holding an intrinsic stretching thermal energy as a sub-Planckian thermal container.

That interval was firstly defined and analyzed according to a correlation function that states the invariant measure of the rate in which the arc element evolves,

$$\langle 0| g_{\mu\nu} d\,\langle\forall|\, x-\varepsilon\rangle_\mu d\,\langle\forall|\, x-\varepsilon\rangle_\nu\, |0\rangle =$$

$$= -d\,\langle\forall|\, t-\varepsilon\rangle_0^2 + R^2_{\langle\forall|\, t-\varepsilon\rangle_0} d\,\langle\forall|\, \vec{x}-\vec{\varepsilon}\rangle\, d\,\langle\forall|\, \vec{x}-\vec{\varepsilon}\rangle\,, \qquad (4)$$

which can obviously be replicated for any cosmology. The dynamics of the continuous expansion of space-time at any scale, working by differential operations on intervals, identifies a physical feature of creative fluxion in real physical regions demarcated by the brackets which are, so to speak, "fluxionant" per se, not dependent on cosmologies.

As the soliton warps the space-time, we may interpret the $\widetilde{\mathrm{X}}^j$ as the transformation components coupled to the metric. The terms in $\widetilde{\mathrm{X}}^j$ provide information about the way in which the metric distorts by the solitary wave.

In the context of warp-drives, the energy of space-time expansion could bring remarkable theoretical support to interstellar transport engineering in the future. For instance, at the junction of an Alcubierre warp bubble with the external space-time, the null-diagonal metric field matrix ζ_{ij} includes the corresponding shape function and has components ζ_{i4} of the form $-4E^{-1}\sqrt{1-e^{2\phi}}$. The components ζ_{i4}

of the metric field matrix,

$$\zeta_{lk} = \begin{bmatrix} 0 & 2E^{-1} & 2E^{-1} & -4E^{-1}\sqrt{1-e^{2\phi}} \\ 2E^{-1} & 0 & 2E^{-1} & -4E^{-1}\sqrt{1-e^{2\phi}} \\ 2E^{-1} & 2E^{-1} & 0 & -4E^{-1}\sqrt{1-e^{2\phi}} \\ 2E^{-1} & 2E^{-1} & 2E^{-1} & 0 \end{bmatrix}, \tag{5}$$

show how the expansion energy at the junction inflects the shape function.

When dealing with the sub-Planckian structure of the inherently expanding geodesic, it may be convenient to work with the energy density associated with the expansion. Assuming that the expansion energy density slowly decreases with time as the universe expands, we can consider it "temporarily" constant in each epoch, and, definitely constant in a sub-Planckian interval of each epoch.

The expansion energy density

Once the nature of the expansion of space-time is understood, we can get a representative expression for the expansion energy — or dark energy — *via* energy density.

According to the purpose of this study, let us concentrate on an inhomogeneous cosmology. Standard LT metric space provides a geometry embedded in a universe supposedly inhomogeneously filled with pressureless dust, a type of incompressible gaseous fluid. This assumption is not grounded in perturbative relations to any FLRW cosmology. Assuming spherical symmetry, that is, $(r \in]0,\infty[, \theta \in]0,\pi[, \phi \in [0,2\pi[)$, the associated arc element is given by

$$ds^2 = -dt^2 + \frac{R'(r,t)^2 dr^2}{1+f(r)} + R(r,t)^2 d\widehat{\Omega}^2, \tag{6}$$

with $d\widehat{\Omega}^2 = d\theta^2 + \sin^2\theta d\phi^2$ and $\{r,\theta,\phi\}$ synchronous-comoving with matter $(dx^i/dt = 0, i = 1,2,3,4)$. In a static LT universe, $R = R_0(r)$ is independent of time, and geodesic equation (1) matches the FLRW limit case where $f(r) = -kr^2$ (this observation is only to emphasize the fact that the deductive reasoning that led to equation (1) remains the same).

A suggestion in progress is to take the continuity equation in LT cosmology and combine it with the state equation of an exotic background Chaplygin gas, supposing the universe not dominated by a cosmological constant (for an assessment of the constraints arising from SNIa, please see references [4] and [5]). Then, in a basic hydrodynamic scheme, we may consider the field equation

$$\ddot{\Xi} + \frac{3}{4}p(t)\Xi = 0, \tag{7}$$

with inhomogeneous solution

$$\Xi = \hat{a}(r)f(t) + b(r)g(t), \tag{8}$$

where $\hat{a}(r)$ and $b(r)$ are arbitrary functions, $f(t)$ and $g(t)$ are two independent particular solutions. Since $\mathcal{B} = \mathcal{B}(r) \equiv \frac{\hat{a}'(r)}{b'(r)}$, and $\alpha \equiv \frac{\hat{a}(r)}{b(r)}$, we can express the fluid expansion as

$$\begin{aligned} \theta &= \partial_t \left(\ln\left[\left(\alpha f + g\right)\left(\mathcal{B} f + g\right)\right]\right) = \\ &= \frac{\left(\alpha \dot{f} + \dot{g}\right)\left(\mathcal{B} f + g\right)}{\left(\alpha f + g\right)\left(\mathcal{B} f + g\right)} + \frac{\left(\alpha f + g\right)\left(\mathcal{B} \dot{f} + \dot{g}\right)}{\left(\alpha f + g\right)\left(\mathcal{B} f + g\right)} = \\ &= \frac{\alpha \dot{f} + \dot{g}}{\alpha f + g} + \frac{\mathcal{B}\dot{f} + \dot{g}}{\mathcal{B} f + g}, \end{aligned}$$

with pressure and energy density respectively

$$p = -\frac{4}{3}\frac{\ddot{f}}{f} \tag{9}$$

and

$$\rho = \frac{4}{3}\frac{\alpha\mathcal{B}\dot{f}^2 + (\alpha + \mathcal{B})\,\dot{f}\dot{g} + \dot{g}^2}{\alpha\mathcal{B}f^2 + (\alpha + \mathcal{B})\,fg + g^2}. \tag{10}$$

Namely, the continuity equation in LT,

$$\dot{\rho} + (\rho + p)\,\theta = 0, \tag{11}$$

and Chaplygin state equation,

$$p = -\frac{\mathfrak{A}}{\rho}, (\mathfrak{A} = \text{positive const.}) \tag{12}$$

where $\mathfrak{A}$ is a positive constant, can be combined, so that

$$\dot{\rho} + \left(-\frac{\mathfrak{A}}{p} + p\right)\left(\frac{\alpha\dot{f} + \dot{g}}{\alpha f + g} + \frac{\mathcal{B}\dot{f} + \dot{g}}{\mathcal{B}f + g}\right) = 0. \tag{13}$$

In fact, equation (12) is a particular case of $p = -\mathfrak{A}/\rho^{\mathrm{a}}$ for $\mathrm{a} = 1$, understanding that in cosmology most likely $0 \le \mathrm{a} \le 1$.

Getting equation (9), continuity equation takes the form

$$\dot{\rho} + \left(\frac{9f^2\mathfrak{A} - 16\ddot{f}^2}{12\ddot{f}f}\right)\left(\frac{\alpha\dot{f} + \dot{g}}{\alpha f + g} + \frac{\mathcal{B}\dot{f} + \dot{g}}{\mathcal{B}f + g}\right) = 0;$$

$$\dot{\rho} = \left(\frac{16\ddot{f}^2 - 9f^2\mathfrak{A}}{12\ddot{f}f}\right)\left(\frac{\alpha\dot{f} + \dot{g}}{\alpha f + g} + \frac{\mathcal{B}\dot{f} + \dot{g}}{\mathcal{B}f + g}\right). \tag{14}$$

We can thus evaluate the real temporal variation of the expansion energy density as the universe expands significantly on large scales. One of the motivations for believing in the equation of state (12) and for accommodating an additional feature to LT cosmology by introducing a Chaplygin gas is the possibility of unification of the dark universe (dark matter and dark energy), filling the cosmos with an inhomogeneous fluid. However, the idea needs to be exhaustively tested and eventually corroborated by future surveys, but I equally believe that we need to take a new look at cosmology beyond the standard model.

Briefly, dark energy as expansion energy of the space-time continuum is independent of the cosmological model, although the acceleration of expansion may vary from region to region, characterizing inhomogeneities in the universe. Clearly, since everything is ultimately made of space-time, powerful explosions like SNIa affect the acceleration of expansion in their surroundings, even if minimally. Thus, an inhomogeneous cosmology must take into account uncertainties in the magnitude of SNIa when crossed with redshift due to that interaction, even if conventional standardization using a single lightcurve fitter is applied.

Modeling SNIa data

Let us move on to modeling SNIa data in accordance with the most relevant studies on the LT metric, for which Einstein's tensor assumes the form

$$G_{\mu\nu} = \left\{\begin{array}{cccc} \frac{\left(R\dot{R}^2 - 2ERc^{-2}\right)'}{R^2R'} & 0 & 0 & 0 \\ 0 & -\frac{(R')^2\left(-2\ddot{R}R - \dot{R}^2 + 2Ec^{-2}\right)}{R^2\left(1+2Ec^{-2}\right)} & 0 & 0 \\ 0 & 0 & \frac{R}{2R'}\left(-2\ddot{R}R - \dot{R}^2 + 2Ec^{-2}\right)' & 0 \\ 0 & 0 & 0 & E_{\theta\theta}\sin^2\Theta \end{array}\right\}.$$

A lot of information about SNIa supported my research, such as that available in the references [3], [19], [22], [25] and [29]. All plots referring to SNIa were performed on the Union 2 data consisting of 557 supernovae. The Union 2 compilation includes a number of refinements in handling of systematic errors, and a SNIa sample over the redshift interval $0.1 < z < 0.3$ (always in red) that has been little scrutinized in the past.

Remember that, in general, an LT model depends on three arbitrary functions, $M(r)$, $\beta(r)$ e $\mathfrak{f}(r)$. The arbitrariness of the function $M(r)$ is, in reality, a result of the freedom of choice of the coordinate system. The function $\beta(r)$, the time of the Big-Bang, must be assumed constant if we imagine an inhomogeneous model in which there is an outer region defined as a critical limit perfectly equivalent to a FLRW universe, thus configuring a Big-Bang universally simultaneous. We can also fix the temporal coordinate t of the constant time hypersurface "now" such that it is equal to the age of the universe t_0 in the FLRW model with $\Omega_0 = 1$, a fact that gives us an extra degree of freedom in the sense that the age of the universe depends on the position. The third function, $\mathfrak{f}$, called "curvature", is an unknown function to be defined in the calculations.

As in reference [6], I will introduce the following three quantities:

$$R(r,t) = a(r,t)r, \tag{15}$$

$$\mathfrak{f} = A(r)r^2, \tag{16}$$

$$M = \beta r^3, \tag{17}$$

where $M(r)$ and $\beta(r)$ (the Big-Bang time) are two arbitrary functions of the co-moving coordinate r to be defined, and a the scale factor. On the other hand, Einstein's equations provide

$$\dot{R}^2 = \mathfrak{f} + \frac{M}{R}, \tag{18}$$

whereby, making substitutions based on the quantities defined above, with $\dot{R} = \dot{a}r$, we obtain

$$\dot{a}^2 r^2 = Ar^2 + \frac{\beta r^3}{ar}, \tag{19}$$

$$\dot{a}^2 = A + \frac{\beta}{a}. \tag{20}$$

The FRW cosmology defined by $A = 0$ takes the standard form $a = t^{2/3}$ for $\beta = 4/9$. This is verified by taking the solution of equation FRW with $t_0 = 0$

$$t - \beta = \int_0^a \frac{du}{\sqrt{A + \frac{\beta}{u}}} \tag{21}$$

$$= \int_0^a \frac{du}{2/3\sqrt{u}} \tag{22}$$

$$= \frac{3}{2}\int_0^a u^{1/2} du \tag{23}$$

$$= \frac{3}{2}\frac{u^{3/2}}{3/2} \tag{24}$$

$$= a^{3/2}. \tag{25}$$

Let us note that, due to the freedom in dealing with r, β could also be a function that converges to 4/9 at infinity. In the assumed LT model, according to the choice of A and β,

$$\Omega_M = \left(1 + \frac{a}{\beta}\right)^{-1}, \tag{26}$$

$$1 + \frac{a}{\beta} = \Omega_M^{-1}, \tag{27}$$

$$a = \beta\left(\Omega_M^{-1} - 1\right). \tag{28}$$

The function A [3], one of the three specializations of the LT model, was arbitrated in the form

$$A = \frac{1}{1 + (\mathrm{c}r)^2}, \tag{29}$$

with c representing a constant of adjustment of the model to the observations (do not confuse this "c" with "c" of the speed of light).

The adaptability of the LT cosmology allows us to return to equation (14) and combine the arbitrary functions explained to obtain a specific expression of the energy density time derivative. We can assume, for example, $\hat{a}(r) = \beta \to 4/9$ and $b(r) = M(r) = \beta r^3 \to 4r^3/9$, from where

$$\alpha = \frac{1}{r^3}$$

and

$$\mathcal{B} = 0.$$

Therefore,

$$\dot{\rho} = \left(\frac{16\ddot{f}^2 - 9f^2\mathfrak{A}}{12\ddot{f}f}\right)\left(\frac{\dot{f} + r^3\dot{g}}{f + r^3 g} + \frac{\dot{g}}{g}\right). \tag{30}$$

At this point, to test the theory, it is interesting to find the luminosity distance, d_L[2], as a function of redshift. It would help a lot to know how each of the quantities involved in the calculation varies as we go deeper into the past through the light cone. We define the null vector

$$\bar{v}_{(a)} = -\partial_t a + \frac{\sqrt{1+\mathfrak{f}}}{R'(r,t)}\partial_r a, \tag{31}$$

in such a way that, from the null geodesic expression, $dt = -\frac{R'(r,t)}{\sqrt{1+\mathfrak{f}}}dr$, we obtain

$$\bar{v}_{(a)}\frac{dz}{da} = \frac{\dot{R}'(r,t)}{R'(r,t)}\left(1+z\right). \tag{32}$$

Since d_L is a function of z and $R(r,t)$, it is also useful to know how $R(r,t)$ varies along the light cone, i.e.

$$\bar{v}_{(a)}\frac{dR(r,t)}{da} = \sqrt{1+\mathfrak{f}} - \dot{R}(r,t). \tag{33}$$

Finally,

$$\frac{dR(r,t)}{da}\frac{da}{dz} = \frac{R'(r,t)}{\left(1+z\right)\dot{R}'(r,t)}\left(\sqrt{1+\mathfrak{f}} - \dot{R}(r,t)\right), \tag{34}$$

$$\frac{da}{dz} = \frac{1}{\left(1+z\right)\dot{R}'(r,t)}R'(r,t)\bar{v}_{(a)} = \frac{1}{\left(1+z\right)\dot{R}'(r,t)}\left(\sqrt{1+\mathfrak{f}}a' - \dot{a}R'(r,t)\right). \tag{35}$$

Thus, we have a system of two differential equations to be integrated to obtain the luminosity distance,

$$\frac{dR(r,t)}{dz} = \frac{R'(r,t)}{\left(1+z\right)\dot{R}'(r,t)}\left(\sqrt{1+\mathfrak{f}} - \dot{R}(r,t)\right), \tag{36a}$$

$$\frac{da(r,t)}{dz} = \frac{1}{\left(1+z\right)\dot{R}'(r,t)}\left(\sqrt{1+\mathfrak{f}}a' - \dot{a}R'(r,t)\right). \tag{36b}$$

Following the model performed by Garfinkle (private communication), for the plots in Figures 3, 4 and 8, I used the "effective

[2] The luminosity distance, d_L, is calculated from the radiation flux l_0 emitted by the source and measured by the observer. Formally, it is given by $l_0 = L_e/4\pi d_L^2$, where L_e is the absolute luminosity of the source measured in its reference frame. In practice, obtaining the luminosity distance of a supernova from observing its light curve requires a set of assumptions. In general, observations are directed to the closest supernovae to establish relationships between color, shape of the light curve in multiple bands, and peak luminosity, since closer objects can be observed at a greater number of bands than more distant objects. Finally, the resulting method of converting light curves to luminosity distances is assumed to apply to all redshifts.

magnitude" according to Perlmutter et al [3], defined as the "effective magnitude in the B band in the rest frame". The latter comes down to the expression

$$\mathrm{m}_B^{\mathrm{eff}} = \mathrm{m}_R - A_R^{\mathrm{MW}} - K_{BR} - \Delta_{B,1.1}, \tag{37}$$

which is essentially the apparent magnitude of the supernova in the R band, corrected under the extinction of the Milky Way by the term A_R^{MW}, converted to the magnitude in the B band in the reference frame in rest by K_{BR} (correction K)[3], and finally corrected depending on the shape of the light curve by $\Delta_{B,1.1}$. Residual errors

[3] Comparing theory with observational data requires meticulous work. There are many causes of noise in observation results. For example, a sample of supernovae at small or large redshifts may be skewed toward the brightest region of any distribution of detected magnitudes. The problem is that there is the so-called *Malmquist bias*, which summarizes the simple fact that intrinsically bright objects are easier to see than intrinsically dim ones. In particular, under the influence of gravitational lensing, and when it comes to strong lensing, the image will certainly be brighter than the source, so, results substantially different from reality can be reached. Furthermore, such an effect on remote SNIa tends to lead to an overestimated calculation of Ω_M. On the other hand, there is a need to establish comparative references in order to establish patterns that make it possible to standardize data or translate them into more appropriate contexts. In this last approach, the so-called "K correction"appears.

The K correction "corrects" for the fact that light sources at different redshifts are generally compared to standards or to each other at different wavelengths at rest frames. Technically, this means that the "K" correction corrects an observation made in a passband to another, or to bolometric values. For any source, the correction K, K_{QR}, is given by the equality

$$\mathrm{m}_R = M_Q + DM + K_{QR}, \tag{38}$$

where M_Q is the absolute magnitude of the source, DM is the distance *modulus* defined by

$$DM = 5\log_{10}\left[\frac{d_L}{10\mathrm{pc}}\right], \tag{39}$$

with $1pc = 3.086\text{x}10^{16}m$. The apparent magnitude m_R of the source is related to its flux spectral density $f_\nu(\nu)$ (energy per unit time per unit area per unit frequency) through the expression

$$\mathrm{m}_R = -2,5\log_{10}\left[\frac{\int_0^\infty \frac{d\nu_o}{\nu_o} f_\nu(\nu_o)R(\nu_o)}{\int_0^\infty \frac{d\nu_o}{\nu_o} g_\nu^R(\nu_o)R(\nu_o)}\right], \tag{40}$$

in which the integration is taken over the observed frequencies ν_o; $g_\nu^R(\nu_o)$ is the flux spectral density for the zero-magnitude or standard source, which, for magnitudes relative to the star Vega, is Vega itself; $R(\nu_o)$, called the "response function", describes the band selected by the analysis device. The absolute magnitude M_Q is defined as the apparent magnitude that the source would have if it were 10 pc away, at rest (i.e., not redshifted). It is related to the spectral density of the luminosity $L_\nu(\nu)$ of the source (energy per unit of time per unit of frequency) according to the equation

$$M_Q = -2,5\log_{10}\left[\frac{\int_{\nu_1}^{\nu_2} \frac{d\nu_e}{\nu_e} \frac{L_\nu(\nu_e)}{4\pi(10\mathrm{pc})^2} Q(\nu_e)}{\int_{\nu_1}^{\nu_2} \frac{d\nu_e}{\nu_e} g_\nu^Q(\nu_e)Q(\nu_e)}\right], \tag{41}$$

in wavelength were compensated by the K correction. Perlmutter defines the effective magnitude as m= $M_B + 5\log(H_0 d_L)$, where M_B is the absolute magnitude in the B band taken at the maximum of the curve of light and d_L is the luminosity distance obtained from the integration of the system of differential equations 36a and 36b. It is important to emphasize that the most distant supernovae are those with the greatest intrinsic brightness.

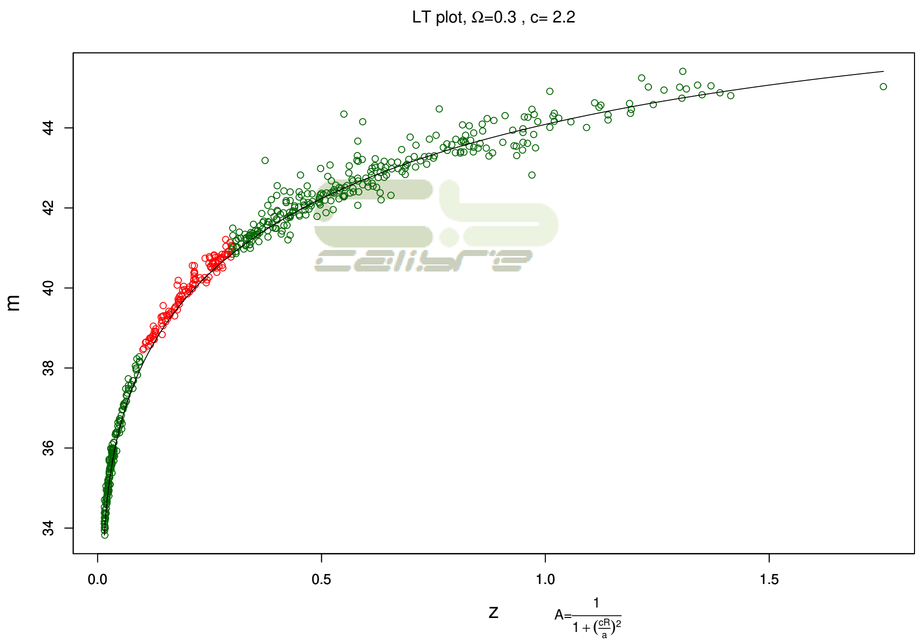

Figure 3: *LT theoretical curve of effective magnitude versus redshift and supernova data according to the Union2 table. The integration of the system of differential equations described previously provides the luminosity distance applied in the effective magnitude formula for the LT cosmological model, say*

$$\left.\begin{array}{l} \dfrac{dR}{dz} = \dfrac{R'}{(1+z)\,\dot{R}'}\left(\sqrt{1+f} - \dot{R}\right) \\ \dfrac{da}{dz} = \dfrac{1}{(1+z)\,\dot{R}'}\left(\sqrt{1+f}a' - \dot{a}R'\right) \end{array}\right\} \Rightarrow d_L\,(z) \Rightarrow \mathrm{m} = M_B + 5\log\left(H_0 d_L\right).$$

in which $Q(\nu_e)$ is equivalent to $R(\nu_e)$, however, in the selected band Q, with the integrals being taken over the emitted frequencies ν_e. Note that M_Q is a bolometric quantity, while m_R is taken in a single band.

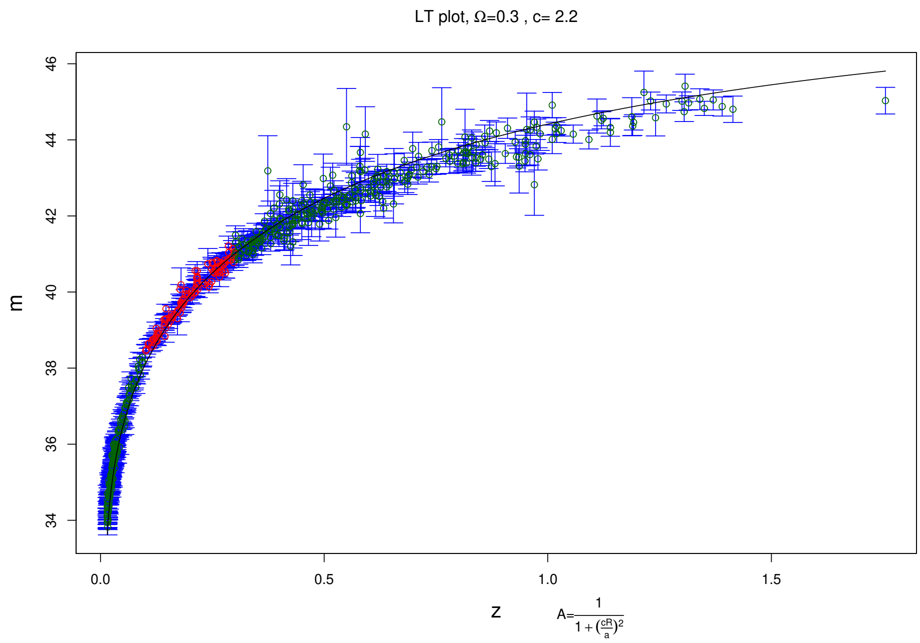

Figure 4: *LT theoretical curve of effective magnitude versus redshift and supernova data according to the Union2 table, including error bars.*

Modeling clusters of galaxies with LT cosmology

On the scale of tens of megaparsecs (clusters and voids), the LT model is very functional, since its evolution is governed by gravity alone. For density prospects beyond the linear regime, LT cosmology appears flexible enough to describe voids.

To model galaxy clusters it was necessary to adjust the curve to R-magnitude (note that, previously, I used the effective magnitude), maintaining the same parameterization introduced for SNIa (Figure 5). To verify whether the model would provide a coherent fit with the observational data obtained from the KiDS survey, and having in mind that inhomogeneous cosmological models enable us to describe the evolution of cosmic structures under non-linear regime, I constructed a method based on non-linear regression that performs a type of reverse engineering on the theoretical LT curve, creating a diffusion of the curve and thus forming a non-linear inhomogeneous field of scattering of its original points (Figures 6 and 7).

The result of the overlap of observation and non-linear theory showed a consistent pattern along z, with coherent proportions between recorded data — with the corresponding error bars — and theoretical scattering, something possible only due to the flexibility of LT cosmology due to its arbitrary functions.

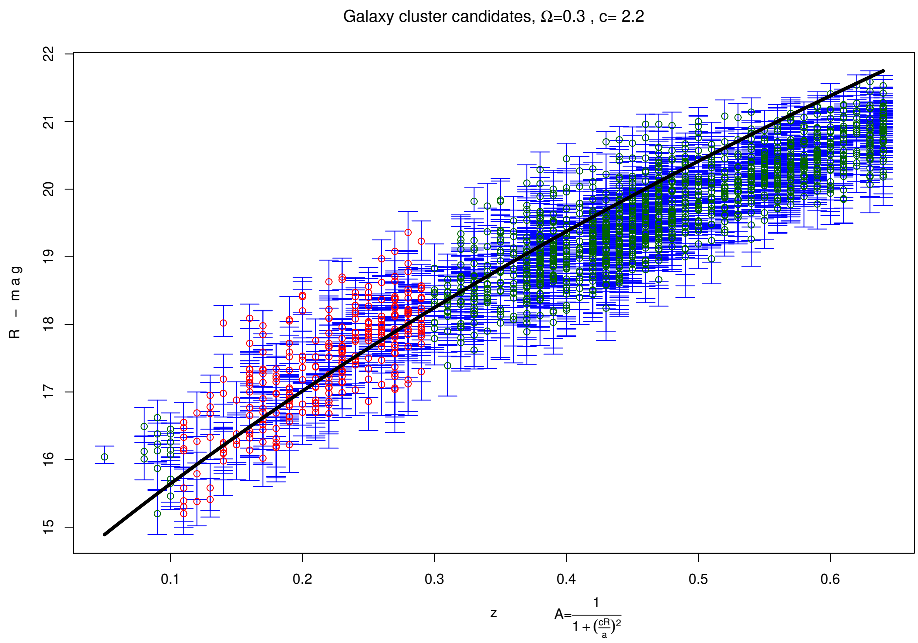

Figure 5: *LT theoretical curve of R-magnitude versus redshift and data of galaxy cluster candidates according to the KiDS survey, including error bars.*

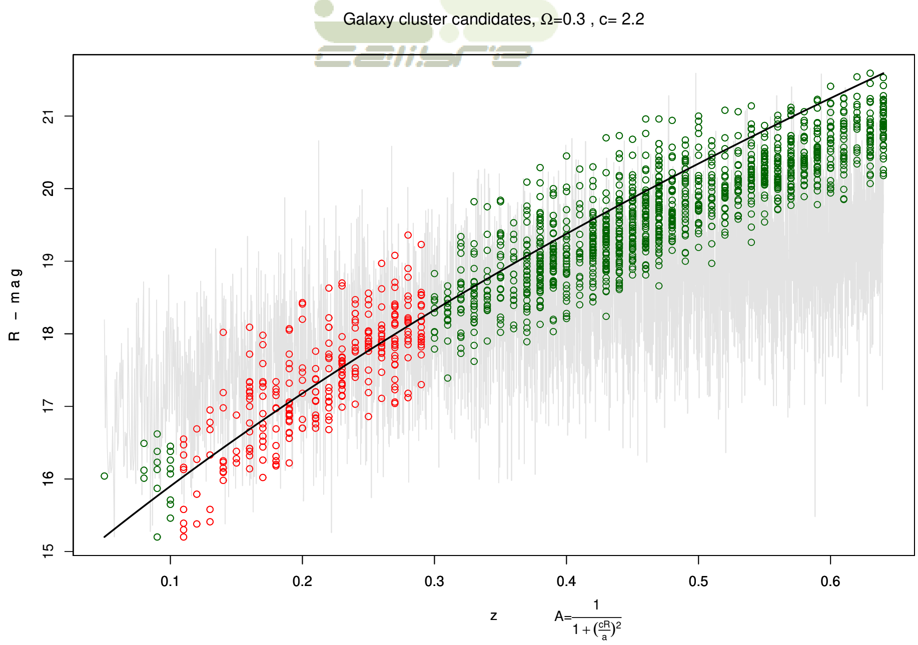

Figure 6: *Scattering based on the ñ-linear regression method applied to the points of the LT theoretical curve (in gray) as a background to the data from KiDS survey.*

Back to the SNIa

The non-linear scattering, which I also call "diffuse reflection" of the theoretical curve, offers a consistent background for the SNIa

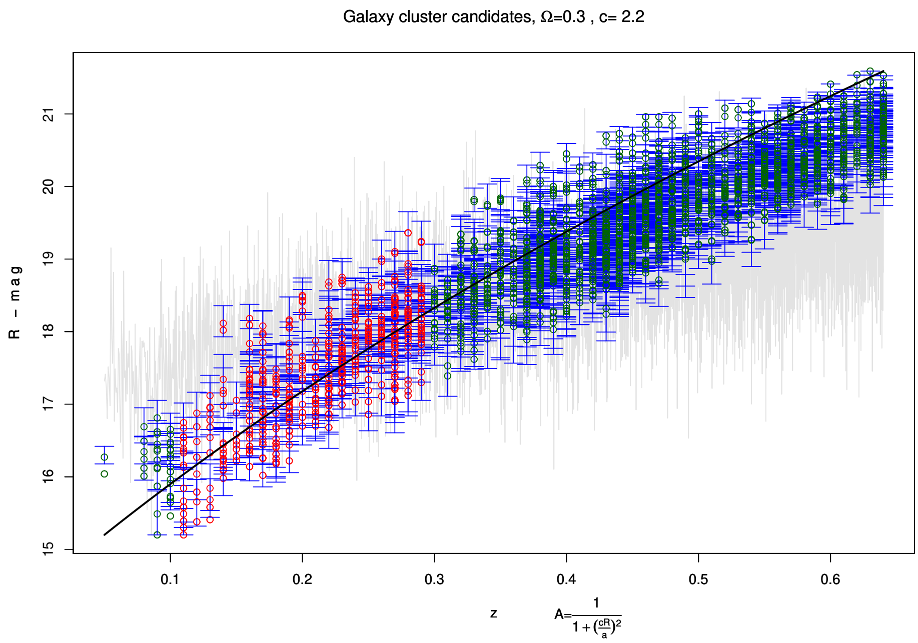

Figure 7: *same scattering and plot with error bars, showing a coherent ratio.*

distribution, as can be seen in the Figure 8. All simulations were reproduced assuming $\Omega_M = 0.3$ and $c = 2.2$, maintaining the best fit to the observational data. What emerges from the analysis of the data is a broad agreement with observation (equatable to the ΛCDM model) at least up to redshifts around 1.8.

Prospects for deeper analysis

Although James Webb still has a lot to offer us in the coming years, there is great expectation regarding the Nancy Grace Roman Space Telescope to be launched by may 2027, with a field of view at least 100 times greater than Hubble, capable of scanning hitherto unprobed regions. It is expected that, among the thousands of exploding stars to be registered, Roman will make it possible to construct a new SNIa survey at z values much higher than those of the first surveys restricted by technological limitations, establishing conditions for precise measurements of how fast the universe is currently expanding, and using SNIa to help understand the nature of dark energy.

But we must be careful with our interpretations of data and resigned to the restrictions of understanding in face of the immensity of the universe and its inexhaustible capacity to surprise us. Heated debates about the value of the expansion energy as dark energy

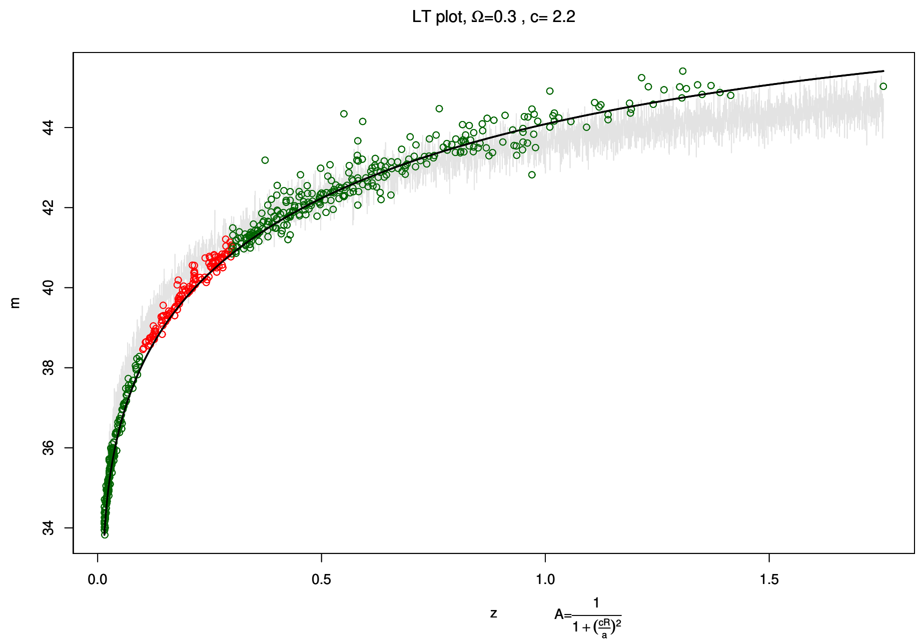

Figure 8: *Non-linear scattering of the LT theoretical curve for SNIa distribution.*

are underway, but I believe it is premature to establish a relationship with the current ideas about cosmological constant. It would be hasty to understand it as such a constant, even though it may be part of its composition.

Indeed, our natural difficulty in dealing with the non-intuitive constantly exposes us to errors in judgment and failures in the criteria for identifying the reasonableness of hypotheses, paradoxically leading us, going against good intuition, to speculate about oddities (see the multiverse hypothesis, for example). For me, in the opposite of the first intuition and considering relativistic modeling, the expansion of the four-dimensional continuum can be thought of in terms of an agent hidden in time — its main fulcrum —, thus being said to be "dark", which manifests itself differently from the phenomena, so to speak, given in Newtonian limits. Note that for time, I used the expression "main fulcrum" of energy, since space and time form an inseparable composite (for a detailed explanation of the space-time composite, please see reference [28]). Simply put, energy is more time than space, and matter is more space than time.

Furthermore and lastly, we need to be attentive. The fundamental problem of physical science as the most penetrating instrument of acquiring knowledge about the world that exists is the seductive trap of the substantiated identity between theoretical model and factual reality. A model is just a representation, a product of the intellect that tries to adjust the observed facts to the human way of explaining things. Because it is a product of the mind of the cognizing self, it will always be a limited and imperfect copy of the material world objectively independent of consciousness. As I hinted earlier, there are deep and fundamental philosophical questions for the future development of cosmology, about which the interested reader will be able to experience interesting insights from the references [13], [14], [16] and [26]. In a nutshell, cosmology, like quantum mechanics, will remain a hatchery of mysteries, uncertainties and exotic things, perhaps refinements of God's humour. It is up to us to appreciate the beauty of such a variety of cosmic events, with the conjecture that perhaps our theories are just tests of the universe trying to understand itself.

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