On Wick-Rotations and Quaternions: The Game of Symmetry between Space and Time

Nilo Serpa

Centro Universitário ICESP, Brazil.
Universidade Santa Úrsula, Rio de Janeiro, Brazil.

Abstract: Despite so many advances, modern physics remains entangled in a number of open questions, some of which are responsible for considerable obstacles in the last 30-40 years. I belong to the group of those who consider that the biggest problem comes from instrumental and conceptual limitations. Our ways of understanding the universe are still extremely limited by the heritage of classical-positivist thinking. Moreover, it is not simple to break the constraints of a brain whose functional design has developed over millions of years in three-dimensional interactive evolution basically conditioned by the imperatives of survival. This natural resistance to a broad theoretical reconstruction leads us to advance very slowly through the innermost essence of the cosmos, having evident reflexes on our motivations and expectations. Present essay examines a preliminary model of spacetime structure in an attempt to offer new conceptual support for the study of the quantum entanglement. Wick-rotations are applied on a quaternionic basis to establish the theoretical foundations of the proposed spacetime symmetries.

Key words: Wick-rotation, quaternion, Clifford algebra, quantum entanglement, spacetime symmetries.

Resumo: Apesar de tantos avanços, a física moderna permanece enredada em várias questões abertas, algumas das quais responsáveis por obstáculos consideráveis nos últimos 30-40 anos. Eu pertenço ao grupo daqueles que consideram que o maior problema vem das limitações instrumentais e conceituais. Nossos modos de entender o universo ainda são extremamente restringidos pela herança do pensamento positivista clássico. Além disso, não é simples quebrar as restrições de um cérebro cujo *design* funcional se desenvolveu ao longo de milhões de anos de evolução interativa tridimensional, basicamente condicionada pelos ditames da sobrevivência. Essa resistência natural a uma ampla reconstrução teórica nos leva a avançar muito lentamente através da essência mais íntima do cosmos, tendo reflexos evidentes em nossas motivações e expectativas. O presente ensaio examina um modelo preliminar de estrutura de espaço-tempo numa tentativa de oferecer novos subsídios conceituais para o estudo do emaranhamento quântico. Rotações de Wick são aplicadas a uma base quaterniônica para estabelecer os fundamentos teóricos da simetria proposta entre espaço e tempo.

Palavras-chave: Rotação de Wick, quatérnio, álgebra de Clifford, emaranhamento quântico, simetrias do espaço-tempo.

Corresponding author: Nilo Serpa, nilo.serpa@icesp.edu.br

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Prologue

The effort to understand the Universe is one of the few things that elevates human life above the level of farce and imprints something of the elevation of tragedy [17] [1].
Steven Weinberg

From the time I went through semiotics as a simple pupil of Umberto Eco [3], I never failed to emphasize the problems of language in science, particularly in physics, and its repercussions in the construction of theories. There is, of course, a philosophical discussion behind linguistic criticism insofar as philosophy is an excellent tool for perfecting our ideas. Adding to all this the understanding of everything that has been said about

[1] Free author's translation.

the dimensionality of the world, about space and about time, I would say that we have at hand an excellent cornucopia of doubts. As Simon Brissenden said recently on the way time is thought after Einstein's relativity,

The language we use and the culture we have been raised in, are profoundly Newtonian and it is difficult to adjust to thinking relativistically [2].

One of the questions that has always instigated me is whether space and time can be put on the same operational footing, that is, whether we can formally work in the same way with both concepts. With respect to the understanding of what time is, although we see in current literature a kind of disguised conformism to the ineluctable difficulty of assimilating it as a real physical quantity, there is no generally agreed upon how to define it rigorously in the same status of the space.

The idea of a complete unity between space and time has long germinated. In fact, it is a time-consuming process of maturation, ranging from the most naive ideas to the less tangible representations of abstract intellection. Hegel, for example, already showed signs of a lucid philosophical perception of reality regarding the apparent separation between space and time:

Sublimiore autem geometria, quae geometriae calculum analyticum jungit, at ex ipsa necessitate temporis spatiique unitorum rationes emetiendi orta est, separationem nonnisi negative per notionem infiniti tollente, neque utriusque veram synthesin proponente, et in negotio suo a formali geometriae et arithmeticae methodo neutiquam discedente [4][1].

He makes clear not only the need for a *spatio-temporal* synthesis, but also the fact that this synthesis had not yet been reached. I think Weil was the one who came very close to do this for the first time in 1922, the year of his seminal work "Space, Time, Matter" [16], although the former serious step toward what might be said the "perfect symmetry" between space and time has been given by Locke, well before Hegel:

[...] expansion and duration do mutually embrace and comprehend each other; every part of space being in every part of duration, and every part of duration in every part of expansion [5].

As much as we think of space and time as inseparable, there remains in our minds a feeling of incompleteness, an intellectual emptiness as to the understanding of the type of symmetry of that combination. This is mainly due to the fact that common sense does not conceive an image by which time and space can be placed under a single broader construct. Even the "amalgam" popularized from Einstein's theory of relativity does not explain a visceral relationship between its constituents (since establishing a pseudo-Pythagorean framework in four coordinates is not in itself sufficient to characterize a perfect symmetry between space and time), or to what extent we can refer to the blend of space and time as not artificial.

Once, a student interested in cosmology and astrophysics asked me if it was possible to travel in time, to which I replied: "If so, we shall need an exotic theoretical model to try to understand how can something transit freely between the ages. We shall probably have to rethink spacetime in the fashion of quantum mechanics, facing paradoxes and restricting our approaches to the limits of information theory" (after all, the idea of transmitting information to the past or to the future is far less pretentious, rather than sending people to Ancient Rome or the Jurassic). I mention this because I think that the shift in time is an excellent paradigm for discussing how space and time could exhibit perfect symmetry. By perfect symmetry I understand the possibility of switching roles played by two interconnected distinct objects.

My research perspective is cosmological and quantum-mechanical, since the theory of quantum spacetime I advocate describes how the cosmic web evolves. There is still much speculation in astrophysical cosmology, even though objects of cosmological importance such as type I-A supernovae remain the subject of extensive studies, as well as the interior of black-holes and the center of spiral galaxies. But, I believe that such speculations, as long as they are inscribed in erudite plausibility, may flourish at some point, whether by observational discoveries or laboratory procedures. It may be that time travel is a matter of perfect symmetry between space and time, if such symmetry exists. In at least one notable instance, it could be true an effect of such a symmetry: the quantum entanglement.

The notion of symmetry is very broad, playing a fundamental role in physics. Particularly, I like the example stemming from Maxwel's remarkable groundbreaking contribution by which one came to understand that a changing electric field generates a magnetic field, and, on the contrary, a changing magnetic field generates an electric field. There is also the beautiful symmetry between bosons and fermions, called supersymmetry, over which I have worked for some years. Supersymmetric theories allowed us to overcome some dichotomies; thanks mainly to supergravity, we obtained the benefit of the symmetry between "matter" (semi-integer spin particles) and "interaction" (integer spin particles). I wonder if space and time would exhibit such a powerful symmetry, capable of manifesting itself through phenomena we do not even suspect.

[1] The superior geometry which results from the fusion of analytic calculus with fundamental geometry, and which arose from the same necessity of measuring the relations of time and space together, overcomes this separation only in a negative way, by means of the notion of infinity, but does not propose the true synthesis of both [...] (free author's translation).

My intention in this essay is to discuss a proposal of spacetime supersymmetry (or perfect symmetry), considering some semantic implementations in the formal language of the theory and specifying under what conditions such supersymmetry could be verified. Whether in the future such representation shall serve as the basis for technological implementations that make some sort of temporal transport possible is a question I do not propose to risk an answer.

I PART: THE WAKE OF THE SCHISM

1 A bit on Wick-rotation

A very puzzle in modern science, while obsequiously left aside, is to find a physical meaning to justify Wick-rotation, a mathematical procedure said by many to be no more than "a mere technical trick". There has been a time when I devoted extensive efforts to the study of the physical significance of Wick-rotation. So, I will allow myself to reproduce a part of my records here, adding some important updates.

The application of Wick-rotation is in such manner confuse that in many works I was even unable to resolve whether the authors were discussing having in mind Lorentzian or Euclidean signature; indeed, I could not see any clear justification with physical significance to introduce imaginary rotations in those discussions. Excepting the few one can find on Wick-rotation applied to the momentum variable k_0 in Green's functions, no remarkable reference has been done, especially about the possible roles fulfilled by bosons and fermions in Wick-rotation (concerning peculiarly the fermions, this was noted also by Nieuwenhuizen and Waldron [**?**]). Even in early quantum physics, the imaginary unit is hollow of physical significance. As an example, concerning Pauli matrices, we see that physicists like to put them in one-to-one correspondence with orthogonal directions in Euclidean 3-space, expressing their orthogonality by the Grassmannian outer product $\sigma_1 \wedge \sigma_2 = \sigma_1\sigma_2 = -\sigma_2\sigma_1$. Thereby, the product $\sigma_1 \wedge \sigma_2 \wedge \sigma_3 = \sigma_1\sigma_2\sigma_3 = i$ reflects the identity between i, as the pseudo-scalar unit for Euclidean 3-space, and a trivector created by the outer product of the orthogonal vectors σ_1, σ_2 and σ_3. Everything is accepted tacitly as a mere formal result. Also in twistor theory, no direct physical interpretation is generally assigned to the complex coordinates. By the way, recalling Clifford algebras, we see that a reflection related to a plane orthogonal to γ^a is given by $\psi \Rightarrow \gamma^a\psi$ in spinor space, and for time-like γ^a we must substitute γ^a by $i\gamma^a$ to satisfy the imposition of identity for squared reflections, indeed a beautiful feature but much more connected to mathematical modeling than to physical requests (I am trying simply to show where we may be more emphatic about physics). Finally, discussing Higgs mechanism for gravity and considering a Lorentz violating spectrum in a model for non-massive gravitons, scientists are once more laconic about the imaginary frequency at very low momenta. After all, a complex number can always be used to register a spinorial association, since the coupling of "i" specifies a twirl,

$$\underbrace{a + \overset{\overset{spin}{\downarrow}}{ib}}_{complex\ number.}$$

Surely, no one else imagines a real rotation in the quantum world. The spin is not a "thing", but only a symbol denoting a type of symmetry belonging to a class of symmetries of which Cartesian rotation is a trivial example.

In 1977 there appeared an interesting work of the French physicist and philosopher of science Jean Émile Charon, *Theorie de la Relativité Complexe*, in which he proposed a complex quadridimensional Riemannian structure to the physical space with a metric

$$Z_{\alpha\beta} = \tilde{Z}_{\alpha\beta} + i\breve{Z}_{\alpha\beta}, Z_{\alpha\beta} = Z_{\beta\alpha}, \tag{1}$$

so that,

$$ds^2 = Z_{\alpha\beta}dy^\alpha dy^\beta, \tag{2}$$

with

$$y^\alpha = \tilde{y}^\alpha + i\breve{y}^\alpha. \tag{3}$$

Charon argues, among other ideas, that only such a complex space turns possible to extend general relativity to quantum field domain and justify the four complex extra components (including time) as a way to assign physical quantities to the theory (for example, the action associated to the spin)[2] . Unfortunately, the theory did not gain the merited attention.

John G. Taylor, at the height of his career, ascribed physical interpretation to imaginary quantities when he wrote the third chapter of "The New Physics", entitled "Faster Than Light". Telling us about Einstein's famous article of 1905, we may read at page 94 of the Spanish version:

... si acelerar una partícula hasta la velocidad de la luz exige una cantidad infinita de energía, acelerarla por encima de este valor requerería una energía imaginaria. Una cantidad imaginaria está formada por el producto de un número real y la raíz cuadrada de menos uno. Aun cuando esta cantidad puede manejarse sin mayor problema como símbolo, en la realidad no resulta posible medirla (Taylor, 1974).

Here, an imaginary physical quantity is one to which there is no sense to apply rules or clocks; is one to which observational operations are not defined. Even so, it is

[2] J. Charon, "*Theorie de la Relativité Complexe*", Albin Michel, Paris (1977).

considerably present within the frame that explains the world.

Also, it is worth remembering the probability distribution in quantum field theory, or, as it is better known, path integral. According to Baez, the amplitudes of probability are proportional to $\exp(-S/i\hbar)$, giving to quantum world an obscure feature if compared to statistical mechanics, where Boltzmann distribution $\propto \exp(-E/T))$ arises naturally from the principle of maximum entropy.

The introduction of the imaginary unit through Wick rotations may be indirectly associated with physical mechanisms of mass acquisition in the supersymmetric relationship between bosons and fermions. Parallel to Higgs's anticipated description of such a mechanism in terms of a boson, I had some interesting insights during my investigations on supergravity.

For now we know only one boson of Higgs, which does not mean that there are no others. Years ago I established a supersymmetric model of gravity with gravitinos confined inside an anti-De Sitter spacetime. The boundary between this spacetime and the De Sitter universe where the gravitational interaction mediators (gravitons) inhabit is actually a field I have called "filtrinic" (relative to a hypothetical particle called "filtrino"), which is the barrier whose "toll" to be paid by an escaping gravitino is its own mass; if "decides" to pay it, gravitino will lose his identity becoming a massless graviton. The filtrino, the hypothetical semi-integer spin particle correlated to the junction field between the two spacetimes, retains gravitino mass, but for a short time as soon as a graviton penetrates the junction and becomes a new gravitino in the anti-De Sitter spacetime. In fact the massive filtrino interacts with the gravitational field through its emissary (graviton) to generate a gravitino. Reverse operation reveals perfect symmetry between graviton and gravitino, since inversely a gravitino interacts with the gravitational field to generate a filtrino. We can see that the junction field is a kind of exotic gravitational region originated from the separation between two fundamentally distinct spacetimes, where the filtrino is an ephemeral fact. For the representation of this mechanism in terms of a spinorial-like matrix formalism, the application of Wick rotations has proved extremely convenient.

Not exactly in the same reasoning line, but in a certain way similar to my above considerations, Nieuwenhuizen and Waldron proposed "a continuous Wick-rotation for Dirac, Majorana and Weyl spinors from Minkowski spacetime to Euclidean space, which treats fermions on the same footing as bosons" [7]. They emphasize that the study does not focuse the Wick-rotation of the momentum variable k_0 but a Wick-rotation of the field theory itself. After some observations, they were led to suggest for a Dirac spinor the Wick-rotation

$$\Psi(\tau, \vec{x}) \to \mathtt{S}(\theta)\Psi_\theta(\tau, \vec{x}), \tag{4}$$

$$\Psi^\dagger(\tau, \vec{x}) \to \Psi_\theta^\dagger(\tau, \vec{x})\mathtt{S}(\theta), \tag{5}$$

in which $\Psi_{\theta=\pi/2} \equiv \Psi_{\mathrm{E}}$ is the Euclidian Dirac spinor and $\mathtt{S}(\theta)$ a diagonal matrix with entries $(e^{\gamma^4\gamma^5\pi/2}, 1, 1, 1)$ that acts only Wick-rotating time sector (the exponents γ^4 and γ^5 are elements of the Euclidean Clifford algebra). Resembling argumentations are applied to Majorana and Weil spinors.

Lately, Matt Visser [15] made an interesting discussion on Wick-rotations in flat spacetimes, recalling the "$i\epsilon$" prescription for quantum field propagators as an elementary consequence of causality in Minkowski space, and showing that the simple procedure of an analytic continuation of the time variable, "$t \to it$" — which agrees only in cases like the Minkowski and Schwarzschild static metrics —, does not offer a consistent generalization for all curved manifolds. In continuation, he obtained an interpretation capable of providing a nice generalization. The reader will find curious the formal coincidence, albeit in very different contexts, between the approach in reference [15] and my approach in terms of a complex "not quite Minkowskian" metric, made below.

In analogous sense, as we may define a dual field ϕ_D and a dual function $F_D(\phi_D)$, so that $\phi_D = F'(\phi)$ and $F'_D(\phi) = -\phi$ constitute a Legendre transformation $F_D(\phi_D) = F(\phi) - \phi\phi_D$ or, which came to be the same, a duality symmetry,

$$\begin{pmatrix} \phi_D \\ \phi \end{pmatrix} \to \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \phi_D \\ \phi \end{pmatrix}, \tag{6}$$

we have in gravitorial theory [11] a symmetry

$$\begin{pmatrix} \boldsymbol{i}_2 \\ \sigma_\eta \end{pmatrix} \to \begin{pmatrix} \gamma_{\overline{11}} & \gamma_{\overline{12}} \\ \gamma_{\overline{21}} & \gamma_{\overline{22}} \end{pmatrix} \begin{pmatrix} \boldsymbol{i}_2 \\ \sigma_\eta \end{pmatrix} \tag{7}$$

for gravitinos or,

$$\begin{pmatrix} \mathbb{1}_2 \\ \sigma_\mu \end{pmatrix} \to \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix} \begin{pmatrix} \mathbb{1}_2 \\ \sigma_\mu \end{pmatrix} \tag{8}$$

for gravitons. In this theory, according to my way of seeing, Wick-rotation is a particular kind of abstract representation that serves very well to explain isometric transformations with physical significance, puting face-to-face, in a simple manner, the state-object and its qualitative change. It preserves a great mathematical inheritance, as we deal with the so intuitive and powerful idea of rotation group but, at the same time, warrants that "something" is no more the "same thing" from the viewpoint of system's physics under isometric isomorphism. For instance, when we Wick-rotate a bosonic representation, we are bringing a fermionic one but in a unique affine frame because $S(g) = iS(g)$.

1.1 Explaining the problem

What must be clear to the reader is that I do not intend to sound identification chords with science fiction ideas (although I appreciate the genre). The discussion here concerns phenomena of the quantum world, having nothing to do with anthropic fantasies; I only wish to work on a conjectural level that may be useful in future works following Popper's thinking, according to which

[...] Risky ideas, unwarranted anticipations, speculative thinking, are the only means we can use to interpret nature: our only "organon", our only instrument to apprehend it. And we must risk ourselves, with these means, to achieve the prize. Those who are not willing to expose their ideas to the eventuality of refutation will not participate in the scientific game[3] [8] (free author's translation).

I also think that space and time in its most intimate connection constitute a phenomenon that transcends any help that perception can give us. Nothing better than an excerpt from Merleau-Ponty (regardless of he may have been confused about some aspects of Einstein's theory of relativity), following Husserl's ideas, to summarize this thought:

[...] every effort to understand the spectacle of the world from within and from the sources demands that we separate ourselves from the effective unfolding of our perceptions and our perception of the world, that we be content with its essence, that we no longer confuse ourselves with the concrete flow of our own life to retrace the whole movement and the main articulations of the world upon which it opens. To reflect is not to match the flow from its source to its last branches; it is to get rid of things, perceptions, the world and perceptions of the world [6] (free author's translation).

My theory starts from two basic questions: is there such a symmetry (a perfect symmetry) between space and time that makes possible information traffic from present to future as well as to past? And if so, is there a phenomenon that insinuates this possibility? According to my interpretation, a perfect symmetry between space and time determines that both can be converted into each other. In other words, the reverse of space is time, and vice-versa. To answer the second question, the only phenomenon we have at hand to help us to justify such a transformation is the quantum entanglement. But, if spacetime can suffer that presumed role reversal, it is reasonable to assume that this reversal begins with quantum entanglement itself, since entangled particles, separated by an arbitrary distance, change their behavior simultaneously as if they broadcasted freely between past and future. The role reversal, I believe, occurs at the moment of quantum entanglement [2].

[3] However, there must be a balance between the need for risk and the usefulness of the risky idea. For example, although we do not have direct access beyond the event horizon, there is no evidence that we should abandon Einstein's equations to describe the interior of a black-hole, at least as long as we keep away from the quantum of spacetime that characterizes its central singularity (as I discussed in another paper, the notion of point has no physical meaning, the reason why it is necessary to specify the quantum of spacetime defined under the action of an intense warp energy).

1.2 The quantum of spacetime

For the discussion that follows, it is interesting to note that the refutation of the old "ether" by the theory of relativity has not eliminated, as many people think, the idea of an absolute space. In fact, space preserves its physical properties, since the equations of motion for material bodies and force fields have not the same form in translational and rotational frames. Thus, what really should be removed from physics is the Newtonian notion of an immovable "container-space" that existed as a frame of reference in absolute rest. This reminder is crucial, because by assigning a sub-Planckian quantum dynamic structure to spacetime, I am postulating its own physical existence.

The ontological limitation of the spirit is remarkable when it comes to describing spacetime; it presents itself sometimes as a fact determined by the mode of relationship between objects, sometimes as a substratum of all that exists. I believe we should embrace both understandings, applying them as required in different thinking situations.

For the first case, if we compare the spacetime in which we live with the spacetime where the entangled particles supposedly live, the following logical reasoning can be supported:

Spacetimes are just states manifested by the ways objects and sets of objects relate to each other; thus, ordinary spacetime, which reflects the observable universe, is the state that results from the way in which the objects we know interconnect, influence, dispose in relation to each other, destroy one another, or merge into new ones. In this everyday spacetime, time is one-dimensional; only future linked to a fixed past.

In Wittgenstein's saying, if more cannot be said, it is better to be silent!

For the second case, let's look at the problem from a physical point of view. In a recent work, I discussed (in French) a question that seems to me to be at the root of the difficulty of reconciling quantum mechanics with general relativity. While cosmology seeks to understand the genesis of the universe, *inter alia*, from an expansion dynamic that, in principle, does not indicate any discontinuous character of spacetime, quantum mechanics reveals the character discrete of the microphysical structure of matter. In my opinion, this discrete image is only what is captured from interactions, and Rovelli is right when he emphasizes interaction as a key concept for understanding the world (note that this conclusion goes well with the previous logical reasoning). But when one speaks of the ultimate tapestry of the universe, of the most intimate content of all matter, it would be a philosophical contradiction to establish an indivisible matter, since the indivisible does not consist in parts, and what has no parts has, theoretically, the nature of a point. But the point is a pure mathematical abstraction; it is now time to remind ourselves that matter is ultimately made up of spacetime itself. So, the most logical way to conceive physical reality is to imagine a dynamic continuum formed by infinitely small expansions or contractions. The dynamic continuum cell, expressed by a quantity as small as you wish, is the quantum of spacetime; no matter how small one can imagine it, it is still smaller, and again smaller, not because it is inaccessible to the rules, but because its dynamics runs in the domain of the infinitely small steps. In this way, the infinitely small translates into spacetime expansions or contractions, as small as one wants, not into abstract intervals of a static geometry. I understand that, most likely, what we call "dark energy" is the energy dissipated by these infinitesimal fluxions. Such an approach, however, is not part of this essay. The fundamentals of the spacetime quantum theory I advocate are found in the references [10] and [12].

Certainly, the human being has no intellectual capacity to fully understand the discussed symmetry, just as the phenomenon of quantum entanglement escapes common reason. It seems even unlikely that we can test such symmetry within the limits of the existence of mankind. It would be necessary, then, to detect some physical phenomenon for which a satisfactory description could be constructed from the proclaimed mutual inversion of roles between space and time.

II PART: QUATERNIONIC MODELING

2 Seeking to model quantum entanglement

Quantum entanglement generates an odd interaction between particles, a kind of linkage that has no equivalent in classical world. Two ways are possible to understand entanglement: 1) from quantum computing, introducing an imaginary quantumfold, a onefold that only exists in quantum descriptions of nature, from which we may obtain an imaginary representation of entangled states by tensor operations applied on it (this quantumfold is not geometrically thinkable, so that it is covered by a special and unique tensor product to one qubit)[4]. To extract real entangled states from this type of coverage we logically need one imaginary gate; 2) from supersymmetry, swiveling spacetime to change the roles of the components of the coordinate basis. I will deal with the second option, as I have considered the first in a previous work [13]. Let's think for a moment about the meaning of attaching the imaginary unit to any quantity. We may suggest that we are applying a marker on a variable that can not be measured (but yes, on which we can virtually and artificially lead a counting). This seems to be the case with time; we can count it abstractly with a clock or an hourglass. We cann't, however, measure it in the strict empirical sense of putting an interval of duration next to a ruler; it does not exist the perception of "spacing" with respect to time.

[4] In fact, entangled particles respond to each other at the same time, being separated only in ordinary space, but solidly bound, embedding with one another in the quantumfold.

Now, suppose that the physics as close as you want to a pair of entangled particles can be described from a four-dimensional coordinate system similar to the Minkowskian system. Let's go back a little to discuss the pure coordinate basis including the markers that will be coupled later to variables, assigning to them temporal or spatial signature. Before an observation (in the experimental sense of the word), there is no logical reason to attribute spatial or temporal signature to a given value belonging to a *Gedankenexperiment* in that vicinity. All we can do is to manipulate the basis by some algebra and see how the markers transform by the algebra. The possible transformations are the symmetries of the theory.

We are next bound to ask: what is the three-dimensional time? It would be more natural to conceive it in two dimensions: past and future! However, a little theoretical acuity makes us realize that the present is also a dimension if we understand that, in a causal line, it is a variable that is very committed to the future and, retroactively, to the past. The present, this fleeting flash, contains the state of the observer, a moving horizon of reference. A curious illustration would be as follows: if we look at the sun, we will have a picture of the past; in the opposite direction, it can be said that the sunlight which has just passed us, going into the future, gives us the past of the future from the wall of a building we are watching, illuminated by that light, that is, the future of the building with a certain "discount". But we are "the present" as observers, and we can shift this present to see a farther past of the sun and a near future of the building (or a shorter separation between building's past and its future), even though the system as a whole head always to the future. Past, present and future are then very dynamic and intertwined! This may sound unintuitive, but contemporary physics is very filled with nonintuitive things. A categorical example of the presence of unintuitive elements at the heart of the fundamental principles of one of the most important scientific theories ever is the trajectory of a light ray. According to Einstein's theory of relativity, following Bertrand Russell's excellent approach (2018), light describes trajectories from minimal paths, which means that between two neighboring points of a light ray the distance is zero (null geodesic) [9]. Still, there is a temporal difference between the two points, such that there are intervals closer to the source than others (all this, and more precisely the concept of null geodesic, in full agreement with my quantum theory of spacetime). This is a logical result of the conception of spacetime, from which space and time no longer separate as in Newtonian physics.

2.1 Wick-rotating quaternions

As suggested earlier, it can be said that one of the gaps in current mathematical physics is the attribution of a direct physical meaning associated with the imaginary unit. In addition to the meaning discussed from expressions (7) and (8), there is a more effective way of introducing the imaginary unit as a physical marker. Particularly, I will propose a tentative model to deal with the hypothetical perfect symmetry between space and time, in which such a direct physical meaning is attributed to the coupling of the imaginary unit.

We consider a base with four coordinates (X_0, X_1, X_2, X_3) with an *a priori* condition: as close as you want to the pair of entangled particles at the moment of its creation, you do not know what coordinates are, whether timelike or spacelike; those coordinates form a quaternion. A *CIS*-temporal quaternion (C) is Minkowskian, that is, constituted by a temporal component and three spatial components; a *TRANS*-temporal quaternion (T) is formed by three temporal coordinates and one spatial coordinate. For simplicity, I call *CT*-quaternion the pair (C, T). Also for simplicity and to avoid confusion, I call "fourth dimension" the component that unidimensionally characterizes space or time. Since I will deal only with the transformations of the quaternionic spacetime base, exemplifying with 2X2 abstract matrix representations, I will adopt arbitrary arrangements constructed from Pauli matrices, which will certainly be useful in further studies.

Firstly, before going on to investigate quaternion transformations, we assume that operators i, j, k map reals to imaginary numbers; the polarity of the i, j, k-axis can be freely fixed according to whether Wick-rotation comes from i, j, k rotating in clockwise or anticlockwise direction. In addition, we must remember that mathematical properties of rotational operators both in 2D and 3D systems are invariant over changes of axial parity [14]. Lastly, imaginary time is a concept that plays crucial role in our understanding of exotic phenomena; Thus, it looks promising to expand the concept to try to explain a phenomenon so exotic as the quantum entanglement.

Now, let the *TRANS*-quaternion coordinates base be equal to $\left(\mathbb{1}, i\sigma_{(k)2}, j\sigma_1, k\sigma_1\right)$, where i, j, k are imaginary units, $\mathbb{1}$ is the unitary matrix 2X2, $\mathbb{O}$ is the zero-matrix 2X2, and $\sigma_{\dots}$ are the Pauli matrices,

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$

having in mind that it makes no difference if we use k or j instead of i, and

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

remebering that to define upon a particular 4-dimensional configuration of co-ordinate axes a system of rotational operators which has the property of being non-commutative leads us to the following relationships between the operators:

$$\begin{aligned} &ij = k; ji = -k; \\ &jk = i; kj = -i; \\ &ki = j; ik = -j. \end{aligned}$$

So, the transformation

$$\begin{pmatrix} \mathbb{O} & \begin{vmatrix} 0 & +k \\ -k & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \begin{vmatrix} +j & 0 \\ 0 & +j \end{vmatrix} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & \begin{vmatrix} +k & 0 \\ 0 & +k \end{vmatrix} \\ \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix} \begin{pmatrix} +\mathbb{1}_2 \\ +i\sigma_{(k)2} \\ +j\sigma_1 \\ +k\sigma_1 \end{pmatrix} = \begin{pmatrix} +\imath\mathbf{i}_2 \\ -\sigma_1 \\ -\sigma_1 \\ -\sigma_1 \end{pmatrix}, \tag{9}$$

is a Wick-rotation of the *TRANS*-quaternion giving a *CIS*-quaternion. Pauli matrices are always useful, because they generate the observable subspace of the two-dimensional Hilbert space and generate transformations in the sense of Lie algebras. We can build several quaternions of this type using Pauli matrices and thus find all matrices that perform Wick-rotations on them. For instance,

$$\begin{pmatrix} \mathbb{O} & \begin{vmatrix} 0 & +1 \\ +1 & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \begin{vmatrix} +j & 0 \\ 0 & +j \end{vmatrix} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & \begin{vmatrix} +k & 0 \\ 0 & +k \end{vmatrix} \\ \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix} \begin{pmatrix} +\mathbb{1}_2 \\ +i\sigma_1 \\ +j\sigma_1 \\ +k\sigma_3 \end{pmatrix} = \begin{pmatrix} +\imath\mathbf{i}_2 \\ -\sigma_1 \\ -\sigma_3 \\ -\sigma_1 \end{pmatrix}. \tag{10}$$

Clearly however, these transformations may be non-trivial, depending on the representation employed. A Wick-rotation in the broad sense is the transformation of a real component into an imaginary one (or *vice-versa*), not necessarily in unambiguous and direct way. The main advantage of the quaternionic marker base is that we can associate it with any magnitude we wish to analyze spatially or temporally.

Wick-rotation turns spatial markers into temporal markers, and contrariwise. Under these circumstances, the previous temporal marker became spatial, restricting space freedom to a "filamentary" structure. What does that really mean? It is reasonable to think that each locus on the observer's filamentary trajectory constitutes its proper space and carries a temporal "trihedron" receiving signals of separate time events occurring at different locations. On the contrary, each moment in the observer's timeline constitutes its proper time and carries a space trihedron receiving signals from separate space events occurring at different times. All this has nothing to do with the accessibility of science fiction films, but it may indicate the possibility of free information transmission from the future to the past. The spatial filament could even be infinite, binding both entangled particles to any distance you want. Note that this has nothing to do with Newtonian-Euclidean view of simultaneity, for there is a proper time (as well as a proper space) of the observer, and the times and spaces relative to each observed event.

Despite everything I said, an overly anthropic universe has dominated our models of reality, leaving us confused by exotic phenomena such as quantum entanglement. The fact is: *a priori*, we may not realize something that does not happen entirely in our testable universe; to assimilate entanglement, a new configuration of spacetime coordinates must be shared by the entangled particles with a fifth connection-component embedded in one of the four-dimensional continuum components. This is precisely the situation in which we need to look for new ways of relating objects quite different from those we usually observe.

2.2 The matrices: complex open algebras

From the diversity of possible combinations, we may restrict the choices only to a set of matrices like matrix (10) (each of them multiplied by its transconjugated gives the unitary matrix) that elegantly represented those base transformations. For instance,

$$\begin{pmatrix} \mathbb{O} & \begin{vmatrix} 0 & +1 \\ +1 & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \begin{vmatrix} +j & 0 \\ 0 & +j \end{vmatrix} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & \begin{vmatrix} +k & 0 \\ 0 & +k \end{vmatrix} \\ \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix} \times$$

$$\begin{pmatrix} \mathbb{O} & \mathbb{O} & \mathbb{O} & \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & +1 \\ +1 & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \begin{vmatrix} -j & 0 \\ 0 & -j \end{vmatrix} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \begin{vmatrix} -k & 0 \\ 0 & -k \end{vmatrix} & \mathbb{O} \end{pmatrix} = \mathbb{1}_4$$

These matrices form what I called a "complex open algebra" (the term "open algebra" was used for the freedom of choice of representations according to the intended physical-mathematical scope). They can also specify an additional invariant symmetry from multiplication operation; thus, it is possible to arbitrarily choose matrices that construct this symmetry. For example, the product of two matrices of a given complex open algebra will be a matrix whose nonzero entries are arranged in the same manner, with the same conjugate disposition. So,

$$\begin{pmatrix} \mathbb{O} & \begin{vmatrix} 0 & +1 \\ +1 & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \begin{vmatrix} +j & 0 \\ 0 & +j \end{vmatrix} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & \begin{vmatrix} +k & 0 \\ 0 & +k \end{vmatrix} \\ \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix} \times$$

$$\begin{pmatrix} \mathbb{O} & \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \begin{vmatrix} 0 & +i \\ -i & 0 \end{vmatrix} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & \begin{vmatrix} -j & 0 \\ 0 & -j \end{vmatrix} \\ \begin{vmatrix} -j & 0 \\ 0 & +j \end{vmatrix} & \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix} =$$

$$\begin{pmatrix} \mathbb{O} & \mathbb{O} & \begin{vmatrix} -i & 0 \\ 0 & +i \end{vmatrix} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & \begin{vmatrix} +1 & 0 \\ 0 & +1 \end{vmatrix} \\ \begin{vmatrix} +i & 0 \\ 0 & -i \end{vmatrix} & \mathbb{O} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \begin{vmatrix} +1 & 0 \\ 0 & +1 \end{vmatrix} & \mathbb{O} & \mathbb{O} \end{pmatrix},$$

where

$$\begin{pmatrix} \mathbb{O} & \begin{vmatrix} 0 & -1 \\ -1 & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \begin{vmatrix} 0 & +i \\ -i & 0 \end{vmatrix} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & \begin{vmatrix} -j & 0 \\ 0 & -j \end{vmatrix} \\ \begin{vmatrix} -j & 0 \\ 0 & +j \end{vmatrix} & \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix} \begin{pmatrix} +\imath\mathbf{i}_2 \\ -\sigma_1 \\ -\sigma_3 \\ -\sigma_1 \end{pmatrix} =$$

$$\begin{pmatrix} +\mathbb{1}_2 \\ +i\sigma_1 \\ +j\sigma_1 \\ +k\sigma_3 \end{pmatrix}$$

is a Wick-rotation of

$$\begin{pmatrix} +\imath\mathbf{i}_2 \\ -\sigma_1 \\ -\sigma_3 \\ -\sigma_1 \end{pmatrix}.$$

In this case, it would be required that all product-matrices of the particular complex open algebra would have the form

$$\begin{pmatrix} 0 & 0 & m & 0 \\ 0 & 0 & 0 & n \\ m^{\dagger} & 0 & 0 & 0 \\ 0 & n & 0 & 0 \end{pmatrix}.$$

2.3 Another dimension?

Of course, if we think of space (or time) reduced to a filament it will be natural to deduce that there may be many independent, but somehow connected, filaments. To assume by the conventional way a fifth dimension that answers by such connection would lead us back to the old question of why we do not perceive it. In addition, we would add an unnecessary matrix complication. It seems more elegant to suppose a hidden component[5] embedded in the fourth dimension and signed by an imaginary unit distinct from the others, defined as $\varepsilon = \sqrt[4]{-1}$. We do not perceive it because it is part of the fourth dimension, and because it is, by definition, null under Wick-rotation. In fact, the component associated with the fourth dimension, that is, the "bridge" between the infinite filaments of time or space, expresses what really constitutes the edifice of the continuum: four-dimensional slices that stand out from a "fifth direction" that is neither time nor space. And, more precisely, the qualitative difference between space and time, which is imposed on us by perception, does not belong to the objective world, as Weil has pointed out [16].

That fifth direction manifests, by definition, the interesting property of being null under Wick-rotations and "sluepotent". An sluepotent operation is one where a quantity a is such that $aa = i$. Thus,

1. $i\varepsilon = j\varepsilon = k\varepsilon = 0;$ (11)
2. $\varepsilon\varepsilon = i.$ (12)

Given this new perspective, let's look at the matrix structure of the representation outlined above. I must define the following matrices,

$$\mathrm{E}_1 = \begin{bmatrix} \sqrt[4]{-1} & 0 \\ 0 & \sqrt[4]{-1} \end{bmatrix} = \begin{bmatrix} \varepsilon & 0 \\ 0 & \varepsilon \end{bmatrix};$$

$$\mathrm{E}_2 = \begin{bmatrix} 0 & -\sqrt[4]{-1} \\ -\sqrt[4]{-1} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\varepsilon \\ -\varepsilon & 0 \end{bmatrix};$$

$$\mathrm{E}_3 = \begin{bmatrix} 0 & -\sqrt[4]{-1} \\ \sqrt[4]{-1} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\varepsilon \\ \varepsilon & 0 \end{bmatrix};$$

$$\mathrm{E}_4 = \begin{bmatrix} \sqrt[4]{-1} & 0 \\ 0 & -\sqrt[4]{-1} \end{bmatrix} = \begin{bmatrix} \varepsilon & 0 \\ 0 & -\varepsilon \end{bmatrix}.$$

From the new definitions and with the bridge represented in the first quaternion component, it is possible to write the transformation

$$\begin{pmatrix} \mathbb{O} & \begin{vmatrix} 0 & -i \\ -i & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \begin{vmatrix} +j & 0 \\ 0 & +j \end{vmatrix} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & \begin{vmatrix} +k & 0 \\ 0 & +k \end{vmatrix} \\ \begin{vmatrix} 0 & -\varepsilon \\ \varepsilon & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix} \begin{pmatrix} (\mathbb{1}_2, \mathrm{E}_1) \\ +i\sigma_1 \\ +j\sigma_1 \\ +k\sigma_3 \end{pmatrix} =$$

$$\begin{pmatrix} +\mathbb{1}_2 \\ -\sigma_1 \\ -\sigma_3 \\ (\sigma_2, \mathrm{E}_3) \end{pmatrix}, \tag{13}$$

where the new quaternion includes the bridge in the last component.

We note that the inversion of this transformation is given by

$$\begin{pmatrix} \mathbb{O} & \begin{vmatrix} 0 & -1-\varepsilon \\ -1-\varepsilon & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \begin{vmatrix} 0 & +i \\ -i & 0 \end{vmatrix} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & \begin{vmatrix} -k & 0 \\ 0 & +k \end{vmatrix} \\ \begin{vmatrix} +k & 0 \\ 0 & -k \end{vmatrix} & \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix} \times$$

$$\begin{pmatrix} +\mathbb{1}_2 \\ -\sigma_1 \\ -\sigma_3 \\ (\sigma_2, \mathrm{E}_3) \end{pmatrix} =$$

$$\begin{pmatrix} (\mathbb{1}_2, \mathrm{E}_1) \\ +i\sigma_1 \\ +j\sigma_1 \\ +k\sigma_3 \end{pmatrix}. \tag{14}$$

We can go on and complete the calculation doing the product of the two matrices

$$\begin{pmatrix} \mathbb{O} & \begin{vmatrix} 0 & -i \\ -i & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \begin{vmatrix} +j & 0 \\ 0 & +j \end{vmatrix} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & \begin{vmatrix} +k & 0 \\ 0 & +k \end{vmatrix} \\ \begin{vmatrix} 0 & -\varepsilon \\ \varepsilon & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix} \times$$

[5] The discussion of transempirical or hidden features in the structure of the universe is not new. Bohm's conjecture about hidden variables dates from the 1950s [1]. Nevertheless, the situation I am discussing does not have exactly the same epistemological nature. In Bohm's model, the well-defined momentum and position variables of a particle are called its supplementary, or hidden, variables with respect to quantum theory; they have simultaneous existence, but they do not explicitly appear in the process except in the form of a certain quantum potential. In my theory, by contrast, there is a hidden supplementary dimension which has, yet, no classical root or any natural correspondence with previously known quantities that would now be simply "unseen" due to new theoretical dispositions and formal arrangements.

$$\begin{pmatrix} \mathbb{O} & \begin{vmatrix} 0 & -1-\varepsilon \\ -1-\varepsilon & 0 \end{vmatrix} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \begin{vmatrix} 0 & +i \\ -i & 0 \end{vmatrix} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & \begin{vmatrix} -k & 0 \\ 0 & +k \end{vmatrix} \\ \begin{vmatrix} +k & 0 \\ 0 & -k \end{vmatrix} & \mathbb{O} & \mathbb{O} & \mathbb{O} \end{pmatrix} =$$

$$\begin{pmatrix} \mathbb{O} & \mathbb{O} & \begin{vmatrix} -1 & 0 \\ 0 & +1 \end{vmatrix} & \mathbb{O} \\ \mathbb{O} & \mathbb{O} & \mathbb{O} & \begin{vmatrix} -i & 0 \\ 0 & +i \end{vmatrix} \\ \begin{vmatrix} -1 & 0 \\ 0 & +1 \end{vmatrix} & \mathbb{O} & \mathbb{O} & \mathbb{O} \\ \mathbb{O} & \begin{vmatrix} +(\varepsilon+i) & 0 \\ 0 & -(\varepsilon+i) \end{vmatrix} & \mathbb{O} & \mathbb{O} \end{pmatrix} .(15)$$

The product of this matrix by its transpose conjugate provides

$$\begin{pmatrix} 0 & 0 & -\sigma_3 & 0 \\ 0 & 0 & 0 & -i\sigma_3 \\ -\sigma_3 & 0 & 0 & 0 \\ 0 & (i\sigma_3, E_4) & 0 & 0 \end{pmatrix} \times$$

$$\begin{pmatrix} 0 & 0 & -\sigma_3 & 0 \\ 0 & 0 & 0 & (-i\sigma_3, E_4) \\ -\sigma_3 & 0 & 0 & 0 \\ 0 & i\sigma_3 & 0 & 0 \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \tilde{\text{U}} \end{pmatrix}, \tilde{\text{U}} = 1 + i.$$

The choices and the sequence of operations were such that we came to the unitary matrix of the theory, the "Ũ-matrix", that is, a matrix with the last diagonal entry equal to the complex unit, consequence of the fifth dimension embedded in the fourth.

It is not purpose of this essay to exhaust all options that would lead to similar results, but merely to demonstrate one consistent result. Since I am dealing only with the geometric framework of the representation, ongoing studies will further illustrate the usefulness of the quaternionic basis discussed with applications.

PART: FINAL REMARKS

3 Times of a new time

Time really fascinates. As a physicist, I try to ease mental tension by having fun with good sci-fi movies, especially those whose scripts deal with the pitfalls of time. Indeed, they can often provoke interesting perspectives. Recently, my favorite series, Star Treck Discovery, has reaffirmed this truth to me. Fictional extrapolations apart, episode 12 of season 2 shows a Klingon monastery on the remote planet Boreth; a monk, guardian of the "time crystals", voiced a remarkable thought: "when the future becomes the past, the present shall be released". If the subject of the film were the quantum entanglement, no verbal construct could be better to synthesize the essence of the phenomenon.

Tied to a linear time and to a way of thinking about separate things, we can hardly decipher the deepest puzzles of nature. The idea of a three-dimensional time from the perfect symmetry between space and time allows us to think of different ways of organizing and describing the relationships between objects and sets of objects. If we accept this possibility, we will be better able to understand quantum entanglement and other phenomena not yet fully elucidated.

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