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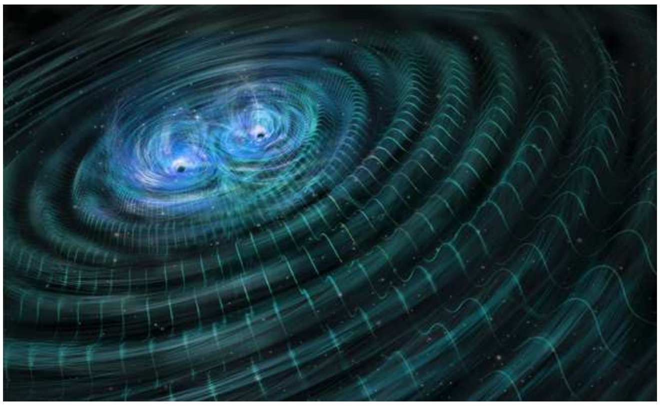

Thesaurus Theoriis Circa Gravitatis et Cætera

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Original research conducted at the Brazilian Center for Physics Research,

fulfilling academic tasks

in post-doctoral character.

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Lecture Notes On Gauge Theories And Inhomogeneous Cosmologies

Quantum Gravity: An Approach From Green's Functions

Nilo Silvio Costa Serpa

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You can not imagine how everything is vague
until try to do it accurately.
Bertrand Russel

Introitus

Thesaurus Theoriis Circa Gravitatis et Cœtera is a compilation subdivided into two chapters, containing the complete research developed during author's post-doctoral period at the Brazilian Center of Physics Research – CBPF, Rio de Janeiro (2016 – 2017). From the heated discussions that took place in a climate of intense knowledge exchange, additions were made to the original text, making it more translucent to the readers.

In the first chapter, the lecture notes related to the Summer Formative Activities – 2017 were meant to compile the main ideas on gauge field theory and modern cosmology, showing the recent contributions of the author in these areas, especially in classical thermodynamics, supergravity and quantum gravity. Although the author has conducted the work without wishing to exhaust the issues in question, this synthesis provides some

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essential formal aspects for further studies, with relevant and updated references, as well as indications of classical readings. Special attention was given to author's application of Lyra's geometry, because of its growing importance in quantum cosmology, and to the so-called «paleogravity», a model of supergravity developed by the author with the purpose to offer a classical representation for supergravity that could be made compatible with the quantum theory of spacetime also developed by himself. Also in general cosmology, the emphasis was on inhomogeneous models because of the debate that they open on the validity of the standard model. In this sense, the author presented his perturbative formalism of weak gravitational lensing by defining an inhomogeneous cosmological refractive index within a Lemaître-Tolman universe. Lastly, some remarks on Stephani cosmology were organized in order to expose the most significant features of this approach. During the reading, it shall be possible to observe some title marks indicating items of interest, as well as small side texts with outgivings of renowned authors.

In the second chapter, the author expanded the discussion about the quantization of spacetime, previously opened, deepening his phenomenological analysis at the surroundings of supermassive bodies with the aid of the formalism of Green's functions. This approach also opens the way for a broader and dialectical debate on quantum gravity and supergravity, seeking to rescue a physical discourse and not merely the math exercise which has been predominant in some research groups.

The author is immensely grateful to the colleagues and external participants for the contributions and affection with which he was received and from which a community of collaborative studies was formed on the most important topics in modern physics. Special thanks goes to our esteemed Professor Helayël, to whom the author directs his highest respects and wishes for a long and brilliant future.

CHAPTER 1

I PART: GAUGE

1 General approach to gauge field theory

In his beautiful book «The Dawning of Gauge Theory» [28] O'Raifeartaigh says that the fundamental idea contained in the gauge symmetry is that if a system remains invariant under a rigid group of continuous transformations (independent of spacetime), then it remains invariant if the group is taken locally (depending on spacetime). Having in mind Lagrangian formalism, we want theories where the Lagrangian density is invariant under internal symmetry transformations that depend on the point in spacetime. If such symmetry implies a dynamic, i.e., a natural description of the appropriate interactions of the theory, then there is a significant gain to the understanding of the physics running in the system under consideration. Thus, the application of the gauge principle consists in the introduction of new fields in the Lagrangian to remove the terms of symmetry breaking of this Lagrangian. This profound principle, which the hard history dates back to the twenties and thirties of the twentieth century, is applied to both quantum and classical situations, being in full compliance with general relativity (GR). Typical continuous transformations applied to quantum fields are unitary transformations of complex phase from which numerical relationships between vectors and operators are kept. So, local symmetries change of phase (rotate) at any angle in the complex plane. In Section **1.2** we shall explain an application of the gauge principle in classical thermodynamics. Since the classical fields of the theory to be presented are complex fields, it seemed quite natural to introduce complex phase transformations in the same spirit as in quantum field theory, even for future quantization of the model if necessary. Throughout the development as it follows, we shall have the opportunity to see how the Lagrangian loses symmetry and how we can restore this symmetry. There are several milestone works on gauge theories (the major of them is in reference [28]), so that our approach emphasizes heuristic and teaching aspects as regards the implementation of a gauge theory. Beginners who wish to learn more about the subject can find a great introduction in «https://terrytao.wordpress.com/2008/09/27/what-is-a-gauge/». A simple and very illustrative work came from Huang [15] where the gauge field is presented as a fiber bundle over spacetime and the gauge vector slides independently along its fiber at each point of spacetime (Figure 1). For physicists more experienced, a good reference is the old book of Narlikar and Padmanabhan [26], containing a complete presentation of the gauge principle.

> *The elevation of the gauge fields to the level of the gravitatinal field is a substantial achievement, but is by no means the end of the story* (O'Raifeartaigh, 1997).

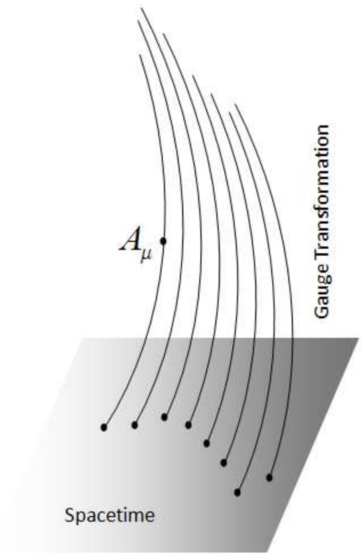

Fig. 1: The fiber bundle of the gauge field.

1.1 A real situation treated by classical field theory

In the aforementioned work, O'Raifeartaigh talks about the difficulties faced at the dawning of gauge theory [28]. From that explanation, adding some enhancements, we can resume the history of gauge theory in four great stages: It is important to note that much of the essential texts on gauge theory remained in German for a long time, and some are still hardly found in translated versions, a fact that has impeded the full access to such documents by the majority of the scientific community not familiarized with the language. For this reason and also because of its distinct theoretical structure, unfortunately Lyra's gauge approach is not commonly treated in the general context of gauge theories, but the cogency of the insights contained in it can not be ignored at all, especially in modern quantum cosmology. Also, in the above stages, indeed very summarized here, there is relatively little contribution from the point of view of the application of gauge theories in classical domain, perhaps because of the illusion that classical physics is a finished discipline (this section shows precisely the opposite). It is true that there is also the fear of polemicize the sacred areas of physics, which is not justifiable since it belongs to the very nature of science the character to be changeable and refineable. Commenting on why Lorenz did not consecrate the relativity of space and time, having done all the legwork, Dirac said

«I think he must have been held back by fears, some kind of inhibition. He was really afraid to venture into entirely new ground, to question ideas which had been accepted from time immemorial» [8].

Only Einstein was able to take the bold leap.

Gauge-dependent quantities can not be predicted, but there is a sense in which they can be measured. They describe «handles» though which systems couple: they represent real relational structures to which the experimentalist has access in measurement by supplying one of the relata in the measurement procedure itself (Rovelli, 2014).

There are scant references on gauge theory applied in classical context other than that of general relativity. In condensed matter, a rare and somewhat dense exception was made by Kleinert [18]. The text as a whole is rather formal and in several sections of the initial part it does not excels in its clarity, but the basic idea is simple. Disregarding quantum effects and analyzing the equilibrium structure of a crystalline material atomic system at zero absolute temperature (a perfectly regular array of atoms known as the «ground state» of the referred system), Kleinert begins to describe the slight shift of that array from a weak perturbation by the corresponding typical «phonons» (the elastic sound waves) of the low excitation states of energy; on one hand, he considers the shift of the array from one layer to another as a gauge transformation on the integer field variables defining the layer; on the other hand, he takes the elastic distortions that may be treated by associated continuous fields that are in fact gauge fields too. The former fields he called «defect gauge fields», while the latter «stress gauge fields» [18]. Obviously the theory itself is quite complex, since the increase in perturbation leads to non-linear terms of the energy expansion become important.

1.2 The classical caloric field gauge approach in practice

To escape a little from the conventional presentations in which monotonously it is repeated the same old examples of classical field theory with not much originality, I shall present a new and realistic context. The case study I shall discuss refers to a feasible project using solar energy in large scales proposed in my doctoral thesis. As we know, solar concentrators have become a reality in the day-to-day response to sanitary and environmental preservation needs [16]. I propose a waste recycling plant as a result of years of research, unifying fundamental issues from field theory and thermodynamics in a comprehensive approach of thermal systems engineering, which is, according to Moran and colleagues, a branch of engineering concerned with how energy is utilized to get benefits in industry, transportation, the daily dealings of home, and so on [25].

Thermodynamics is a beautiful macroscopic theory, built on a few fundamental presuppositions (which makes it more attractive and nice). It describes the effects of macroscopic systems formed by a large number of microscopic entities (spins, molecules, particles, etc.) that obey the basic laws of classical mechanics or quantum mechanics, as the case may be. Analyzing the generality of thermodynamics and its late claim as a solidly established physical science, we can speculate that the prevalence of mechanistic models occurred only by a matter of secular precedence of mechanics and its huge success to explain the world of the immediate things. The completeness of thermodynamics is mainly marked by its evolutionary approach of the physical systems, pointing the entropy as a fundamental variable — defined in a manner somewhat abstract from a variational principle — in the process of evolution. Going from the statement that heat is energy in transit and assuming the thermodynamic equilibrium of the system as the macroscopic state for which the entropy is a maximum, it is possible to realize any physical phenomenon, insofar as the dynamics of the universe is in the end summarized by dissipation and energy exchange processes. Thus, it is also possible to historically understand the almost total lack of application of classical field theories in the context of thermodynamics , except perhaps indirectly in some specific situations where the thermal state of the system appears secondarily in the general analytical framework applied.

My case study shows a consistent application of the classical field theory in thermodynamics, focusing on the subject of recycling condensed matter, specifically in order to establish a system of solid waste treatment. Briefly, any waste fills a prototype system (Figure 2) of two cylindrical graphite chambers in which pyrolysis and recycling processes shall take place. Internally subjected to a vacuum, first chamber (pyrolysis chamber) receives the concentrated sunlight rays from a concave array of mirrors on a quartz window placed at one of the circular bases of the chamber. At high temperatures atomic disruptions produce gases and liquids that flow to the recycling device, inside which the gradient of temperatures $T_1, T_2, T_3, T_4, ..., T_n$ allows to a recovery of products $P_1, P_2, P_3, P_4, ..., P_n$ from the hottest layers to the cooler. A computational control system conducts catalytic agents, whose actions enter the processes associated with temperatures to ensure the outputs of programmed materials, and the recombination of remnant atoms into inert substances in the form of usable waste. To reduce the entropy and expand the productivity of the heat generation we introduced an auxiliary piping system for the laminar flow of a nanofluid to establish a convection process of heat transfer [9,44]. Lastly, products, final residues and usable waste are sent respectively to inventory and appropriate containment, remembering that the so called «pyrolysis ashes» – similar to the dust and blast furnace sludge – which constitute the usable waste can be used in the cement industry. All the energy needed to run the engine is solar, being the possible surplus routed to the public network.

The theoretical model developed treats the thermal energy inside the pyrolysis and recycling chambers as a complex scalar field, the so-called «caloric field» to be measured with precision and controlled at each point of its confinement for a maximum of efficiency in management of byproducts and pyrolysis process. Theoretical basis for the construction of classical fields may be found in the works of Maggiore [23] and Radovanović [31]. In addition, the theory and its application to the power plant forms the econophysical foundations to match operations management and environmental management in a unified operational level just in the sense pointed out by Kurdve et al. [21] to include the waste management supply chain.

Accordingly classical field theory [23], present model supposes a differential polynomial in ξ, the Lagrangian density $\mathcal{L}(\xi)$, given by

$$\mathcal{L} = \left(\partial_q \xi\right)^* \left(\partial^q \xi\right) - |\xi|^2 + 2\gamma^2 |\xi|^2 \ln|\xi|, \tag{1}$$

whose action over a certain region $\mathcal{M}$ in space and time is

$$\mathcal{S}\left(\xi\right) = \int_{\mathcal{M}} \left[\left(\partial_q \xi\right)^* \left(\partial^q \xi\right) - |\xi|^2 + 2\gamma^2 |\xi|^2 \ln|\xi|\right] d\mathcal{V} dt. \tag{2}$$

Here, from my first proposal, ξ represents a scalar complex massless caloric field , $d\mathcal{V}$ an infinitesimal volume of space, dt an infinitesimal time interval, and γ a real scalar to be defined later which depends on the system's environment in question [44]. The caloric field obeys the field equation

$$\partial_q \partial^q \xi + \left(1 - \gamma^2\right) \xi - 2\gamma^2 \xi \ln|\xi| = 0, \tag{3}$$

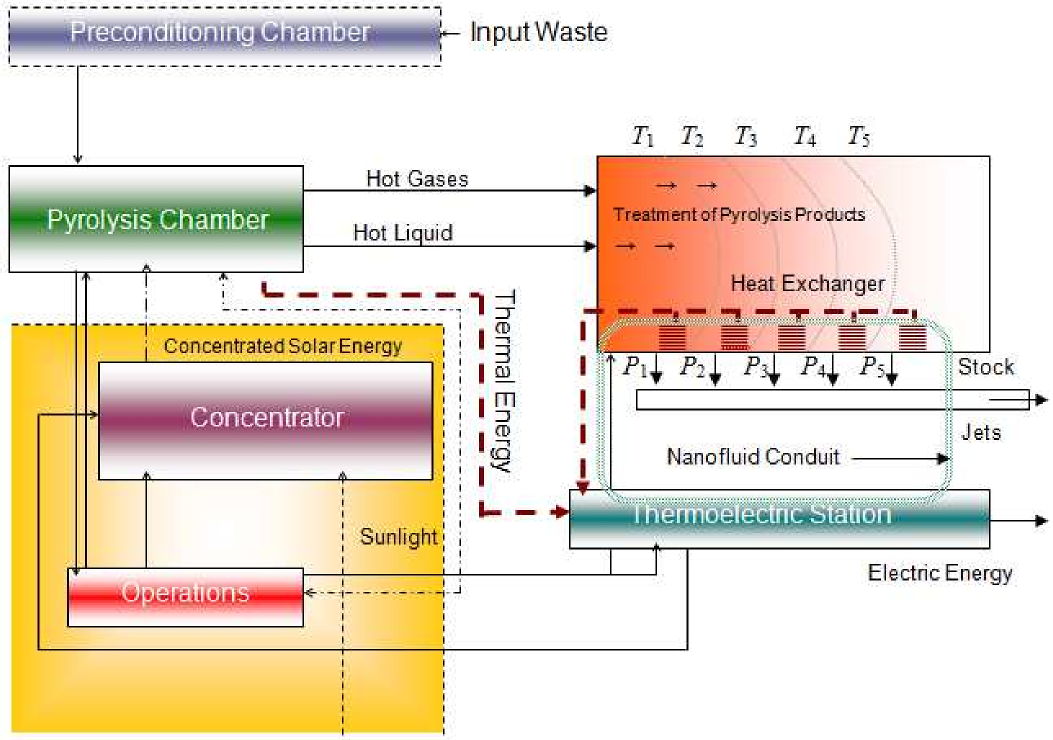

Fig. 2: The complete scheme of the proposed thermodynamic engine. Note the cicle of energy with a thermoelectric station feeding continually the usine as well as it is powered by solar energy (from Serpa's Ph.D. thesis in French [44]).

being the field entropy in generalized coordinates q given by

$$S = \int -2\gamma^2 |\xi|^2 \ln |\xi| dq. \tag{4}$$

Thus, field equation includes an entropy term $-2\gamma^2 \xi \ln |\xi|$ in the dynamics of the field and expression (4) is just a straightforward generalization of Gibbs entropy . It is worth noting that for $|\xi|^2 < 1$ it follows that $2 \ln |\xi| < 0$; thus, $S > 0$ for every non-trivial system state. The factor $(1 - \gamma^2)$ in the second term of equation (3), the so-called «luminothermic capacity», reflects the potential power offered by the natural surroundings. Its action under the field shows how field is influenced by the external conditions. Thus, caloric field equation governs the evolution of the thermal energy field and the corresponding entropy produced.

In a strictly thermodynamic theory, the fields are representations of the energy as heat, while the entropy function is a «potential». Therefore, we mainly deal with heat exchanges which can lead to macroscopic states where interactions between the original field and matter modify the former by the emergence of a gauge field, and thus the establishment of a «massive» factor.

The usual way to present gauge theory begins with the introduction of a global continuous symmetry to the action, say an overall phase. The action does not change if we proceed the transformation $\xi' \to e^{iQ\theta}\xi$. The symmetry group of this transformation is the Lie group $U(\mathbb{1})$. On behalf of discourse economy, now we begin by the introduction of a local phase changing, say

$$\xi' = e^{iQ\theta(q)}\xi$$

and its conjugate

$$\xi^{\dagger'} = e^{-iQ\theta(q)}\xi^{\dagger}.$$

We rewrite our former Lagrangian

$$\mathcal{L}_0 = \partial_q \xi^\dagger \partial^q \xi - |\xi|^2 + \gamma^2 |\xi|^2 \ln |\xi|^2 .$$

A local transformation based on common partial differentiation gives

$$\mathcal{L}_0' = \partial_q \left(e^{-iQ\theta(q)}\xi^\dagger\right) \partial^q \left(e^{iQ\theta(q)}\xi\right) - |\xi|^2 + \gamma^2 |\xi|^2 \ln |\xi|^2 ; \tag{5}$$

$$\mathcal{L}_0' = \left[-iQe^{-iQ\theta(q)}\partial_q\theta(q)\xi^\dagger + e^{-iQ\theta(q)}\partial_q\xi^\dagger\right]\left[iQe^{iQ\theta(q)}\partial^q\theta(q)\xi + e^{iQ\theta(q)}\partial^q\xi\right] - \\ - |\xi|^2 + \gamma^2 |\xi|^2 \ln |\xi|^2 ;$$

$$\mathcal{L}_0' = \boxed{Q^2\partial_q\theta(q)\partial^q\theta(q)\xi^\dagger\xi - iQ\partial_q\theta(q)\xi^\dagger\partial^q\xi + iQ\partial^q\theta(q)\partial_q\xi^\dagger\xi} + \partial_q\xi^\dagger\partial^q\xi - \\ - |\xi|^2 + \gamma^2 |\xi|^2 \ln |\xi|^2 . \tag{6}$$

This operation, as we can see in box, breaks Lagrangian invariance adding the first three terms resulting from the transformation. Consequently, we need another operator, namely the covariant derivative

$$\mathcal{D} = \partial_q - iQA_q \text{ (ou } \mathcal{D} = \partial_q + iQA_q),$$

with the introduction of the gauge field A_q so that we can feature a unitary transformation as

$$\mathcal{L}_0 \xrightarrow{U(\mathbb{1})} \mathcal{L}_0' = (\partial_q + iQA_q)\, e^{-iQ\theta(q)}\xi^\dagger \,(\partial^q - iQA^q)\, e^{iQ\theta(q)}\xi - e^{-iQ\theta(q)}\xi^\dagger e^{iQ\theta(q)}\xi +$$

$$+\gamma^2 e^{-iQ\theta(q)}\xi^\dagger e^{iQ\theta(q)}\xi \ln\left(e^{-iQ\theta(q)}\xi^\dagger e^{iQ\theta(q)}\xi\right){}^{1}. \tag{7}$$

The phases are canceled, leaving the short expression

$$\mathcal{L}_0 \xrightarrow{U(\mathbb{1})} \mathcal{L}'_0 =$$

$$\left(-iQ\partial_q\theta\xi^\dagger + \partial_q\xi^\dagger + iQA_q\xi^\dagger\right)\left(iQ\partial^q\theta\xi + \partial^q\xi - iQA^q\xi\right) - \xi^\dagger\xi + \gamma^2\xi^\dagger\xi\ln\left(\xi^\dagger\xi\right). \tag{8}$$

Making up multiplications term-to-term we obtain

$$\mathcal{L}_0 \xrightarrow{U(\mathbb{1})} \mathfrak{L}'_0 =$$

$$\boxed{Q^2\partial_q\theta\partial^q\theta\xi^\dagger\xi - iQ\partial_q\theta\xi^\dagger\partial^q\xi} - Q^2A^q\partial_q\theta\xi^\dagger\xi +$$

$$\boxed{+iQ\partial^q\theta\partial_q\xi^\dagger\xi} + \partial_q\xi^\dagger\partial^q\xi - iQA^q\partial_q\xi^\dagger\xi -$$

$$-Q^2A_q\partial^q\theta\xi^\dagger\xi + iQA_q\xi^\dagger\partial^q\xi + Q^2A_qA^q\xi^\dagger\xi - \xi^\dagger\xi + \gamma^2\xi^\dagger\xi\ln\left(\xi^\dagger\xi\right), \tag{9}$$

where I kept boxed terms that shall be canceled. However, based on the above development, this cancellation comes from the potential introduction in the expression

$$\mathcal{L}_{Gauge} = -Q^2A^q\partial_q\theta\xi^\dagger\xi - iQA^q\partial_q\xi^\dagger\xi - Q^2A_q\partial^q\theta\xi^\dagger\xi + iQA_q\xi^\dagger\partial^q\xi + Q^2A_qA^q\xi^\dagger\xi, \tag{10}$$

which we called «gauge Lagrangian». Then, we have

$$A'_q = A_q + \partial_q\theta,$$

from which

$$\mathcal{L}'_{Gauge} = -Q^2\left(A^q + \partial^q\theta\right)\partial_q\theta\xi^\dagger\xi - iQ\left(A^q + \partial^q\theta\right)\partial_q\xi^\dagger\xi - Q^2\left(A_q + \partial_q\theta\right)\partial^q\theta\xi^\dagger\xi +$$

$$+iQ\left(A_q + \partial_q\theta\right)\xi^\dagger\partial^q\xi + Q^2\left(A_q + \partial_q\theta\right)\left(A^q + \partial^q\theta\right)\xi^\dagger\xi. \tag{11}$$

The reader must note that the gauge field A_q does not transform by covariant mode. The way it transforms come from the requirement

$$\left(\mathcal{D}_q\xi\right)' = \left(\partial_q - iQA'_q\right)\xi';$$

$$\left(\mathcal{D}_q\xi\right)' = \left(\partial_q - iQA'_q\right)e^{iQ\theta(q)}\xi;$$

$$\left(\mathcal{D}_q\xi\right)' = e^{iQ\theta(q)}\left(\partial_q\xi + iQ\partial_q\theta(q)\xi - iQA'_q\xi\right);$$

$$\left(\mathcal{D}_q\xi\right)' = e^{iQ\theta(q)}\left[\partial_q + iQ\partial_q\theta(q) - iQ\left(A_q + \partial_q\theta(q)\right)\right]\xi;$$

[1] Here I wrote the complete terms with the phases so that the reader realizes that they cancel each other.

$$(\mathcal{D}_q\xi)' = e^{iQ\theta(q)}\left[\partial_q + iQ\partial_q\theta(q) - iQA_q - iQ\partial_q\theta(q)\right]\xi;$$
$$(\mathcal{D}_q\xi)' = e^{iQ\theta(q)}\left(\partial_q - iQA_q\right)\xi.$$

Usually it is assumed that A_q describes some new and independent degrees of freedom of the system. By applying the change, it follows that

$$\mathcal{L}'_{Gauge} = [[-Q^2A^q\partial_q\theta\xi^\dagger\xi - Q^2\partial^q\theta\partial_q\theta\xi^\dagger\xi]] - iQA^q\partial_q\xi^\dagger\xi\boxed{-iQ\partial^q\theta\partial_q\xi^\dagger\xi}+$$
$$[[-Q^2A_q\partial^q\theta\xi^\dagger\xi]]\boxed{-Q^2\partial_q\theta\partial^q\theta\xi^\dagger\xi} + iQA_q\xi^\dagger\partial^q\xi\boxed{+iQ\partial_q\theta\xi^\dagger\partial^q\xi}+$$
$$+Q^2A_qA^q\xi^\dagger\xi\left[\left[+Q^2A_q\partial^q\theta\xi^\dagger\xi + Q^2\partial_q\theta A^q\xi^\dagger\xi + Q^2\partial_q\theta\partial^q\theta\xi^\dagger\xi\right]\right]. \qquad (12)$$

Since the theory is Abelian, the order of the factors in the multiplication does not matter. Terms in double brackets are canceled naturally, while boxed terms cancel the terms of symmetry breaking of the former Lagrangian. So,

$$\mathcal{L}'_0 + \mathcal{L}'_{Gauge} = \partial_q\xi^\dagger\partial^q\xi - |\xi|^2 + \gamma^2|\xi|^2\ln|\xi|^2 +$$
$$+Q^2A_qA^q\xi^\dagger\xi + iQ\left(A_q\xi^\dagger\partial^q\xi - A^q\partial_q\xi^\dagger\xi\right), \qquad (13)$$

or

$$\mathcal{L}'_0 + \mathcal{L}'_{Gauge} = \partial_q\xi^\dagger\partial^q\xi - |\xi|^2 + \gamma^2|\xi|^2\ln|\xi|^2 +$$
$$+Q^2A_qA^q\xi^\dagger\xi + iQ\left\{A_q\partial^q, A^q\partial_q\right\}_{\xi^\dagger,\xi}. \qquad (14)$$

Additional terms that express the interactions between fields carry the generator of the symmetry group of the theory. Since the field A_q is added to our Lagrangian as a tool to assert gauge invariance of the caloric field kinetic term, we must recognize the need to add a kinetic term for the gauge field itself. Thereby, we introduce a field strength tensor, built from the commutator of covariant derivatives

$$[D_p, D_q] = \left[(\partial_q - iQA_q)(\partial_p - iQA_p)\right] - \left[(\partial_p - iQA_p)(\partial_q - iQA_q)\right];$$
$$[D_p, D_q] = \left[\partial_q\partial_p - iQ\partial_qA_p - iQA_q\partial_p - Q^2A_qA_p\right] -$$
$$-\left[\partial_p\partial_q - iQ\partial_pA_q - iQA_p\partial_q - Q^2A_pA_q\right];$$
$$[D_p, D_q] = \left[-iQ\partial_qA_p - iQA_q\partial_p\right] - \left[-iQ\partial_pA_q - iQA_p\partial_q\right];$$
$$[D_p, D_q] = iQ\left(\partial_pA_q - \partial_qA_p\right) = iQ\mathcal{F}_{pq}.$$

The new kinetic term must also preserve Lorentz invariance, so that it assumes the form

$$\mathfrak{F} = \mathcal{F}_{pq}\mathcal{F}^{pq}. \qquad (15)$$

Thus we have a new code for the Lagrangian with embedded transformation of the gauge field, which is

$$\mathfrak{L}_0' = \mathcal{D}_q \xi^{\dagger} \mathcal{D}^q \xi - |\xi|^2 + \gamma^2 |\xi|^2 \ln |\xi|^2 - \mathfrak{F}. \tag{16}$$

Given that the kinetic terms of the classical fields involved do not originate from a mechanical model, there is in principle no reason to assume fractional constants in these terms. Now, the question to ask is: what is the need for a gauge approach to this classic situation in a so familiar terrain like thermodynamics? The answer depends on a correct physical intuition, as on the almost inexhaustible capacity for representation of the physical-mathematical formalism. We have already shown that an unconventional approach to thermal energy is possible. If the massless caloric field, as presented above, was simply generated in a vacuum, nothing new would take place. However, when interacting with the mass of waste, the field generates mass for itself, since the thermo-physical and chemical reactions triggered generate heat providing thermal feedback to the former caloric field, plus a small amount of volatile mass assimilated by the field. This mass is then represented by the constant of minimum coupling with the gauge field, called «Q» (the symmetry group generator), something like a «caloric charge» or better yet «minimal thermal mass factor of dynamic interaction». We note that this corresponding generator does not respect the former vacuum of the chamber.

For me, a gauge theory is any physical theory of a dynamic variable which, at the classical level, may be identified with a connection on a principal bundle (Trautman, 1980).

Thereby, the introduction of the gauge field discovers a new physics, free from derivatives on this field, namely, the interactions that are triggered by the action of the field on the waste which could not exist before introducing the material into the pyrolysis chamber. Indeed, gauge field A_q mediates a «strain»[2] between the fields (and their derivatives) with coupling Q. The symmetry varies point to point, since the processes are subject to a gradient of temperatures and a random volatilization of matter (the phase of the fieldfunction can be chosen arbitrarily at each spacetime point). This information should be part of the stochastic processing algorithm to be initialized in «Operations» (Figure 2) in order to accurately calculate the amount of non recycled material, and the mass percentage assimilated into the field. Lastly, with the recycling of all materials from pyrolysis , the remaining pyrolytic ashes feature a completely inert environment within the chambers. In this situation, the Lagrangian density interactions sector must be annulled.

[2] We can say that a local symmetry generates a «strain» coupled to the «caloric charge». In other words, inside the chamber, if we gauge caloric energy and the minimal thermal mass of dynamic interaction, we shall get forces (internal pressures that can be attributed to the shock of the pyrolytic plasma molecules against the walls of the chamber, thereby being transmitted impulse to the walls) for which the sources are the energy and momentum of the molecules.

Exercise 1.21 *Prove that for a classical complex scalar field ξ, locally defined by a positional parameter $\theta(q)$, we may write*

$$\{A_q\partial^q, A^q\partial_q\}_{\xi^\dagger,\xi} \propto 2iA^q\partial_q\theta(q).$$

Exercise 1.22 *Consider the calorie field*

$$\xi = e^{in\gamma q-\vartheta} \therefore \tag{17}$$

$$\partial^q\xi = in\gamma e^{in\gamma q-\vartheta}, \tag{18}$$

and its conjugate

$$\xi^\dagger = e^{-in\gamma q-\vartheta} \therefore \tag{19}$$

$$\partial_q\xi^\dagger = -in\gamma e^{-in\gamma q-\vartheta}, \tag{20}$$

where n is the polytropic index, γ is the opacity of the medium and ϑ is the refractive index of the focal quartz window [44]. Show, in one dimension, that in the natural gauge ($A^\mu = A_\mu = 1$) the minimal thermal mass factor of dynamic interaction is equal to $2n\gamma$ in the inert state of the pyrolysis chamber.

1.3 The Dirac Lagrangian

The discussion made in the previous section intended to show the most relevant points to be considered when implementing a gauge symmetry, namely the correct perception of the scope of the theory and its suitability to the problem addressed. In the practical case studied, we had as focus the need for a precise knowledge of thermodynamic processes aiming the maximal reduction of the entropy produced in a thermodynamic engineering system to recycling condensed matter. In addition, the program presented in short showed how the gauge theories may be close to our increasingly urgent operational needs.

Similarly to the classical case, we can consider implementing a gauge symmetry from a Lagrangian spinorial structure. The simplest example refers to a unitary transformation of type $UU^\dagger = U^\dagger U = \mathbb{1}$ on spinors, given the Dirac Lagrangian density. For a free particle of mass m we have after Dirac the expression

$$L_0 = \bar{\psi}\left(i\gamma^\mu\partial_\mu - m\right)\psi, \tag{21}$$

where ψ is the Dirac spinor (wave function), $\bar{\psi}$ is the adjunct spinor, and γ^μ is a set of $4X4$ matrixes that defines a Clifford algebra. Again, a local transformation based on common partial differentiation breaks Lagrangian according to

$$
\begin{aligned}
L_0 \xrightarrow{U(\mathbb{1})} L_0' = \bar{\psi}' \left(i\gamma^\mu \partial_\mu - m\right) \psi' \qquad (22)\\
= e^{-iQ\theta} \bar{\psi} \left(i\gamma^\mu \partial_\mu - m\right) e^{iQ\theta} \psi \\
= e^{-iQ\theta} \bar{\psi} \left[i\gamma^\mu \partial_\mu \left(e^{iQ\theta} \psi\right) - m e^{iQ\theta} \psi\right] \\
= e^{-iQ\theta} \bar{\psi} \left\{i\gamma^\mu \left[e^{iQ\theta} \partial_\mu \psi + iQ e^{iQ\theta} \partial_\mu \theta \psi\right] - m e^{iQ\theta} \psi\right\} \\
= \bar{\psi} \left(i\gamma^\mu \partial_\mu - \gamma^\mu Q \partial_\mu \theta - m\right) \psi. \qquad (23)
\end{aligned}
$$

Similarly, through the minimal coupling, we introduce the covariant derivative

$$D_\mu \equiv \partial_\mu + iQA_\mu \tag{24}$$

in such a way that we preserve Lagrangian properties under the local gauge transformation

$$A_\mu \xrightarrow{U(\mathbb{1})} A_\mu' = A_\mu - \partial_\mu \theta. \tag{25}$$

The Abelian intensity tensor is thus defined as

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{26}$$

which remains invariant under the given gauge transformation (for more details, please, see reference [41].

Exercise 1.31 *Write the final expression of the Lagrangian density after the introduction of the covariant derivative and the respective gauge transformation for a complete description of quantum electrodynamics.*

Hint*: consider* $Q\bar{\psi}\gamma^\mu\psi$ *as the electromagnetic 4-current.*

1.4 The Yang-Mills gauge theory

Presently, it is recognized that the most important quantum field theories for describing elementary interactions are gauge theories. It can be said that the most advanced models in this context evolved from the first works of Yang and Mills. The Yang-Mills gauge approach begins at 1952-1954 [57] when they suggested a field similar to the electromagnetic field. As Yang-Mills equations provided the classical description of massless waves that travel at the speed of light, it appeared natural at that moment to try the same approach to describe other forces, mainly the strong interaction binding protons and neutrons into nuclei. However, the massless nature of classical Yang-Mills waves brought serious drawbacks to applying Yang-Mills theory to other forces, since weak and nuclear interactions are short-range forces and many of the associated particles are massive.

As we know, the initial approach of Yang and Mills consists in a non-abelian gauge field theory based on $SU(2)$ symmetry [3]. Protons and neutrons come to be considered nearly identical (if one just concentrates on the nuclear forces ignoring charge), except by the isotopic spin («up» for protons and «down» for neutrons). Since this isotopic spin is a local variable, it can be different for each spacetime point (the isotopic gauge); for instance, the proton up state at one point is not in general the same at any other point. Thereby, just as the electromagnetic potential connects the phase of wavefunctions at different points, there must be an isotopic spin potential connecting states of isotopic spin at different points by rotation of the isotopic spin direction. So, the isotopic spin transforms as

$$\psi' = S(x,t)^{-1}\psi, \tag{27}$$

where $S(x,t)$ is the isotopic spin rotation at a given spacetime point, and ψ is column vetor, the doublet field

$$\psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}$$

Continuing the analogy with electromagnetism, to cancel off the very known extra terms generated by taking the gradient of the potential, it was introduced the covariant derivative written as

$$D = \nabla - i\epsilon A(x,t), \tag{28}$$

where ϵ is the coupling constant. The potential obeys

$$A' = S^{-1}AS + \left(\frac{i}{\epsilon}\right) S^{-1}\nabla S. \tag{29}$$

The nonabelian field strength is given by

$$F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - i\epsilon \left[A_\mu, A_\nu\right], \tag{30}$$

[3] However, the initial expectations were not confirmed, since local $SU(2)$ transformations play no role in strong interactions. Now we understand these forces as governed by an $SU(3)$ gauge theory called quantum chromodynamics (the term was introduced after the word «colour» to be used for the degrees of freedom transforming under $SU(3)$. Lastly, it is important to remark that theories based on $SU(2)$ gauge transformations hold relevance for the weak sector.

which reduces to the form (26) when the gauge fields commutator vanishes. The nonabelian gauge field A_μ matches the complete gauge invariant Lagrangian density

$$L = -\frac{1}{2}\mathrm{Tr}\left(F_{\mu\nu}F^{\mu\nu}\right) + \bar{\psi}\gamma^{\mu}\left(i\partial_\mu - Q\partial_\mu\theta\right) - m\bar{\psi}\psi, \tag{31}$$

which is the sum of a kinetic part with the Dirac Lagrangian for a fermion doublet given by expression (23).

Even though some prospects of the Yang-Mills theory remain out of reach for now, there are studies on the application of $SU(2)$ Yang-Mills fields in cosmology, considering the second and the fourth order terms of the Yang-Mills field strength tensor respectively playing the roles of radiation and cosmological constant [10].

1.5 Superfields and gauge theory

The implementation of superfields aims to facilitate in a remarkable manner the calculations in supersymmetric field theories, from the moment that supersymmetry is identified and established. Everything starts from the fact that in ordinary spacetime supersymmetry is not manifest, being the customary Lagrangian formulation not the most appropriate to model supersymmetric field theories. Therefore, we must consider a superspace, that is, a Minkowski spacetime increased by fermionic 2 + 2 anti-commuting Grassman coordinates $\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$ (associated to de supersymmetry generators $Q_\alpha, \bar{Q}_{\dot{\alpha}}$), forming a new superspace with eight coordinates tagged by $\left(x^\mu, \theta_\alpha, \bar{\theta}_{\dot{\alpha}}\right)$, where the x^μ are the bosonic coordinates with Lorenz vector indice, θ_α are the Grassmanian complex coordinates with left-handed spinor indice and $\bar{\theta}_{\dot{\alpha}}$ their conjugates with right-handed spinor indice. Consequently, superfields are nothing but fields in this superspace, or, which come to be the same, functions of the superspace coordinates which are subject to the translations that characterize supersymmetry, say

$$x^\mu \to x^\mu - i\bar{\vartheta}\bar{\sigma}^\mu\theta - i\vartheta\sigma^\mu\bar{\theta};$$

$$\begin{aligned} \theta_\alpha &\to \theta_\alpha + \vartheta_\alpha; \\ \bar{\theta}_{\dot{\alpha}} &\to \bar{\theta}_{\dot{\alpha}} + \bar{\vartheta}_{\dot{\alpha}}. \end{aligned} \tag{32}$$

where ϑ_α, $\bar{\vartheta}_{\dot{\alpha}}$ are Grassmann spinor parameters. In such a context, nothing could be more natural than talking about a supergauge. Supercovariant derivatives that map superfields to superfields are defined as

$$\mathcal{D}_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial x^\mu}; \tag{33}$$

$$\bar{\mathcal{D}}^{\dot{\alpha}} = \frac{\partial}{\partial \bar{\theta}_{\dot{\alpha}}} + i\bar{\sigma}^{\mu\dot{\alpha}\alpha}\theta_{\alpha}\frac{\partial}{\partial x^{\mu}}. \tag{34}$$

Concluding this brief summary, superfields, while superspace functions, can be understood in terms of expansions in power series in θ_α and $\bar{\theta}_{\dot{\alpha}}$

$$\mathcal{F}\left(x^{\mu}, \theta_{\alpha}, \bar{\theta}_{\dot{\alpha}}\right) = f(x^{\mu}) + \theta_{\alpha}\phi\left(x^{\mu}\right) + \bar{\theta}_{\dot{\alpha}}\bar{\varphi}\left(x^{\mu}\right) + \theta_{\alpha}\theta_{\alpha}m\left(x^{\mu}\right) +$$

$$+\bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\alpha}}n\left(x^{\mu}\right)+\sigma^{\mu}\bar{\theta}_{\dot{\alpha}}v_{\mu}\left(x^{\mu}\right)+\theta_{\alpha}\theta_{\alpha}\bar{\theta}_{\dot{\alpha}}\bar{\lambda}\left(x^{\mu}\right)+\bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\alpha}}\theta_{\alpha}\psi\left(x^{\mu}\right)+\theta_{\alpha}\theta_{\alpha}\bar{\theta}_{\dot{\alpha}}\bar{\theta}_{\dot{\alpha}}d\left(x^{\mu}\right), \tag{35}$$

with component fields $(f(x^{\mu}), \phi\left(x^{\mu}\right), \bar{\varphi}\left(x^{\mu}\right), m\left(x^{\mu}\right) ...)$, and having all higher powers of $\theta_\alpha, \bar{\theta}_{\dot{\alpha}}$ vanished.

Supersymmetry is a beautiful symmetry between bosons and fermions, although there is no evidence of it in Nature. This does not mean that it is not present, but that it must be well hidden (Fayet, 1980).

Exercise 1.51 *Prove that the covariant derivatives $\mathcal{D}_\alpha$ and $\bar{\mathcal{D}}^{\dot{\alpha}}$ anticommute between themselves.*

II PART: GRAVITY

Any cosmological theory is supported by the gravitation theory. Gravity is the only relevant force in the scale of galaxy clusters and beyond. The gravitation theory can be constructed in different ways and this is still a source of puzzles for thoughtful men, mainly in discussions about quantization of gravity and unification of all forces. In fact, there are three main approaches to relativistic gravity theories:

- gravity is a property of spacetime itself, the geometry of curved spacetime;
- gravity is a kind of matter within the spacetime (the relativistic field theory in flat spacetime);
- gravity is the effect of the direct interaction between ponderable particles.

No matter the choice, it is important to look upon that up to now relativistic gravity has been tested experimentally only in weak field approximation. The notes that follow document my studies on gravity under different points of view, which the conciliation, if any, is in the future.

2.1 Paleogravity: from a bit of subversive physics ✓

Supersymmetry (SUSY) is a Bose-Fermi symmetry referring to the spectrum of coupling energy among particles; it is a device that tries to fulfill a phenomenological gap between the sectors of spectrum related to electroweak interactions and GUT scale (from 10^2 Gev to 10^{16} Gev). The gap results from the second Higgs quantization, required in Weinberg-Salam, forcing the introduction of SUSY mechanisms to provide intermediary physics inside those limits. Successive symmetry breaks are in part supplied by gravitic fields that do not couple (at least in thesis) with matter. Supergravity (SUGRA) is the supersymmetry that occurs in gravity. The smallest theory of supergravity relates two types of fields referring to the hypothetical particles graviton and gravitino. The relevance of supergravity to cosmology is that it offers an effective field theory behind the expanding universe and timedependent scalar fields.

Supersymmetry describes fermions and bosons in a unified way as partners of supermultiplets. Such multiplets necessarily have a decomposition in terms of boson and fermion states of different spins. So, the supergravity multiplet consists of the graviton and its superpartner, the gravitino (in fact, the gravitino multiplet contains $(1; 3/2)$ and $(-3/2; -1)$, that is a gravitino and a gauge boson; on the other hand, the graviton multiplet includes $(3/2; 2)$ and $(-2; -3/2)$, corresponding to the graviton and the gravitino). Really the spin 2 graviton derives from the rank 2 of the metric tensor $g_{\mu\nu}$ which describes the gravitational field. At first look, gravitino could have spin 5/2 as often as 3/2, but the advantage to choose spin 3/2 is the absence of the goldstino in supersymmetry breaking theories.

In the model that shall follow, the reader should understand that the terms «graviton» and «gravitino» specify merely the symmetries of the theory and should not be seen as elementary particles in the strict sense. My approach on supergravity consists in a classical framework in the sense that the fields involved do not have, in principle, probabilistic character. As well pointed Rovelli [38], the spatial and temporal features of the gravitational field come to be lost from the moment in which one assumes a granular structure of gravity and so the quantized dynamics of the field with its probabilistic nature. Such a loss would certainly jeopardize any alternative approach wishing to make use of the classical conception of spacetime, even in the particular case of further quantization of the spacetime itself [46]. This approach is associated with the concept of G-closure[4], a type of spacetime bubble whose the internal side would be described by an adS spacetime ($O(3, 2)$ symmetry) dominated by gravitinos embedded into an external dS spacetime dominated by gravitons. The supersymmetric exchange of mass related to the pair graviton/gravitino takes place at the junction between the two spacetimes.

The main restriction on the inclusion of the fourth interaction in the unification process is the fact that the effects of gravity result from a long cumulative process on a large scale. This means that past seems to play an important role in gravity. Still, it is well known that several physical systems can be modeled using differentiable manifolds. In Lagrangian mechanics, for instance, the dynamic equations of a system turn out to be the Euler-Lagrange equations for a defined functional on a given manifold. This formulation is often supported by Riemannian manifolds, and we can see the so familiar principles of conservation as manifestations of invariance of the Lagrangian density in face of a group of smooth transformations, the diffeomorphisms of the manifold. For a nonlocal theory, to which in principle it would not be appropriate to ensure the invariance of the Lagrangian by introducing a covariant derivative, it would be interesting to get a set of diffeomorphisms that could be deducted from the own system's dynamics,

[4] The G-closure shall be seen below and was detailed in reference [46], but it can be understood here as a bubble of inhomogeneity immersed in a FLRW homogeneous spacetime.

thus defining a process of acquisition of mass over time consistent with the establishment of gravitational phenomena at large scale.

So, let us consider a phenomenological Lagrangian density exhibiting a time-integral and something like a «border gauge» field mass-coupled to gravitino[5], such as

$$\mathcal{L} = M^2|g\rangle\langle\breve{G}\rangle\partial_\tau\langle\breve{G}\rangle\int|g\rangle d\tau + 1/3M^2\langle\breve{G}\rangle^3 + i\breve{r}\partial_\tau\breve{r}, \tag{36}$$

where the kets mean that fields are represented with the aid of a math structure called «gravitor»[6]. Gravitors are dual «column-objects» generated from the group $S(\gamma_\eta)$ given by the 2×2 matrices γ_η

$$\begin{pmatrix}0&1\\1&0\end{pmatrix},\begin{pmatrix}0&-i\\i&0\end{pmatrix},\begin{pmatrix}1&0\\0&-1\end{pmatrix},\begin{pmatrix}0&-1\\-1&0\end{pmatrix};$$

$$\begin{pmatrix}0&i\\-i&0\end{pmatrix},\begin{pmatrix}-1&0\\0&1\end{pmatrix},\begin{pmatrix}i&0\\0&i\end{pmatrix},\begin{pmatrix}-i&0\\0&-i\end{pmatrix};$$

$$\begin{pmatrix}1&0\\0&1\end{pmatrix},\begin{pmatrix}-1&0\\0&-1\end{pmatrix},\begin{pmatrix}0&i\\i&0\end{pmatrix},\begin{pmatrix}0&-i\\-i&0\end{pmatrix};$$

$$\begin{pmatrix}0&1\\-1&0\end{pmatrix},\begin{pmatrix}0&-1\\1&0\end{pmatrix},\begin{pmatrix}i&0\\0&-i\end{pmatrix},\begin{pmatrix}-i&0\\0&i\end{pmatrix}.$$

The above referred dual column-objects form the group $\bigcup$ of the gravitors with elements $(\pm\mathbb{1}_2,\gamma_\eta)$ and $(\pm\boldsymbol{i}_2,\gamma_\eta)$. From $\bigcup$ we are interested in the subgroup $\bigcup(\boldsymbol{i}_2)$ of the gravitors that can represent Wick-rotations from one another under the adS Clifford subalgebra $\mathbb{C}_{3,2}^{(\gamma_\mu)}$, so that we have in gravitorial theory a duality symmetry

$$\begin{pmatrix}\boldsymbol{i}_2\\\gamma_\mu\end{pmatrix}\to\begin{pmatrix}\gamma_{11}^- & \gamma_{12}^-\\\gamma_{21}^- & \gamma_{22}^-\end{pmatrix}\begin{pmatrix}\boldsymbol{i}_2\\\gamma_\mu\end{pmatrix} \tag{37}$$

for gravitinos, where γ_{ab}^- is the inverse matrix of γ_{ab}, or,

$$\begin{pmatrix}\mathbb{1}_2\\\gamma_\nu\end{pmatrix}\to\begin{pmatrix}\gamma_{11} & \gamma_{12}\\\gamma_{21} & \gamma_{22}\end{pmatrix}\begin{pmatrix}\mathbb{1}_2\\\gamma_\nu\end{pmatrix} \tag{38}$$

[5] As the question is to describe the influence of the past on a local observation, it would seem contradictory to establish a covariant derivative. So I set out from an integration imposed on the Lagrangian, reversing the approach and making that a transformation rule could arise from the Lagrange equation itself. Although the disregard of inheritance factors is in part consequence of an exaggeration of simplification, non-locality phobia in quantum field theory is very related with the fear to lose Lorentz and gauge invariance, both well preserved with local variables.

[6] In fact, there is another Lagrangian for the interaction between gravitons and gravitinos, but I will limit myself to just discuss the border gauge, suggesting to the reader the reference for more details.

for gravitons. Examples of resulting components for gravitons and gravitinos, accordingly this representation, are respectively:

$$G_\mu = \left[\begin{pmatrix} \mathbb{1}_2 \\ \sigma_1 \end{pmatrix}, \begin{pmatrix} \mathbb{1}_2 \\ i\sigma_2 \end{pmatrix}, \begin{pmatrix} \mathbb{1}_2 \\ i\sigma_3 \end{pmatrix}, \begin{pmatrix} \mathbb{1}_2 \\ \boldsymbol{i}_2 \end{pmatrix} \right], \tag{39}$$

$$g_\mu = \left[\begin{pmatrix} \boldsymbol{i}_2 \\ i\sigma_1 \end{pmatrix}, \begin{pmatrix} \boldsymbol{i}_2 \\ -\sigma_2 \end{pmatrix}, \begin{pmatrix} \boldsymbol{i}_2 \\ -\sigma_3 \end{pmatrix}, \begin{pmatrix} \boldsymbol{i}_2 \\ -\mathbb{1}_2 \end{pmatrix} \right], \tag{40}$$

where

$$\mathbb{1}_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{41}$$

and

$$\boldsymbol{i}_2 = \begin{pmatrix} i & 0 \\ 0 & i \end{pmatrix}, \tag{42}$$

with the customary Pauli matrices,

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{43}$$

Those gravitors were related by the action of the subalgebra $\mathbb{C}_{3,2}^{(\gamma_\mu)}$ according to

$$\begin{pmatrix} 0 & \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \\ \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} & 0 \end{pmatrix} \begin{pmatrix} \mathbb{1}_2 \\ i\sigma_3 \end{pmatrix} = \begin{pmatrix} \boldsymbol{i}_2 \\ -\sigma_3 \end{pmatrix};$$

$$\begin{pmatrix} 0 & \begin{vmatrix} 0 & -i \\ i & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & i \\ -i & 0 \end{vmatrix} & 0 \end{pmatrix} \begin{pmatrix} \mathbb{1}_2 \\ i\sigma_2 \end{pmatrix} = \begin{pmatrix} \boldsymbol{i}_2 \\ -\sigma_2 \end{pmatrix};$$

$$\begin{pmatrix} \begin{vmatrix} i & 0 \\ 0 & i \end{vmatrix} & 0 \\ 0 & \begin{vmatrix} i & 0 \\ 0 & i \end{vmatrix} \end{pmatrix} \begin{pmatrix} \mathbb{1}_2 \\ \boldsymbol{i}_2 \end{pmatrix} = \begin{pmatrix} \boldsymbol{i}_2 \\ -\mathbb{1}_2 \end{pmatrix};$$

$$\begin{pmatrix} 0 & \begin{vmatrix} 0 & i \\ i & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & i \\ i & 0 \end{vmatrix} & 0 \end{pmatrix} \begin{pmatrix} \mathbb{1}_2 \\ \sigma_1 \end{pmatrix} = \begin{pmatrix} \boldsymbol{i}_2 \\ i\sigma_1 \end{pmatrix}.$$

A peculiar thing about gravitors is that they are multiplied by each other via direct product of their two matrix components, while the action of Clifford algebra is a normal matrix product. So,

$$\begin{pmatrix} \boldsymbol{i}_2 \\ \sigma_3 \end{pmatrix}^2 = \begin{pmatrix} \boldsymbol{i}_2 \\ \sigma_3 \end{pmatrix} \circledast \begin{pmatrix} \boldsymbol{i}_2 \\ \sigma_3 \end{pmatrix} = \begin{pmatrix} \boldsymbol{i}_2 \times \boldsymbol{i}_2 \\ \sigma_3 \times \sigma_3 \end{pmatrix}.$$

Now, a Lagrangian model that includes a time integral on the gravitino field as described above, I call «paleogravity». I implemented this way because it is expected that light gravitinos of mass $\lesssim \mathbf{O}(10)eV$ may contribute appreciably to the total matter of the universe, affecting structure formation since early epochs to the present days [29]. I suppose the states of graviton are «mirrored» in states of gravitino, always in pairs, beneath adS Clifford algebra[7]. The fields $\langle G|$ and $|g\rangle$, as coordinates of the whole system, are related to gravitons and gravitinos respectively. The field $\langle \breve{G}\rangle$ is the gravitor inscription of the mass retained at the adS zone with M^2 appearing due to $\langle \breve{G}\rangle$ and its coupling to other fields. The field $\breve{r}$ is an auxiliary non-coupled field defined at the junction between the two spacetimes. Time integrals applied denote strong interference of system's history on local field inhomogeneities. From Euler equation applied to this Lagrangian density, we get

$$\frac{d}{d\tau}\left\{M^2|g\rangle\langle \breve{G}\rangle \int |g\rangle d\tau\right\} - M^2|g\rangle\partial_\tau\langle \breve{G}\rangle \int |g\rangle d\tau - M^2\langle \breve{G}\rangle^2 = 0; \quad (44)$$

$$\langle \breve{G}\rangle = |g\rangle^2 + \partial_\tau|g\rangle \int |g\rangle d\tau. \quad (45)$$

In my theory, the field $\langle \breve{G}\rangle$ is in fact a transformation of the gravitino field according to non-local contributions. Therefore, one can use expression (45) to impose an integro-differential constraint on any field or set of fields in order to preserve Lagrangian symmetry. For the sake of brevity, we may put $\langle \breve{G}\rangle = \mathrm{A}'$ and $|g\rangle = \mathrm{A}$ without loss of generality, so that, for a given manifold $\mathbb{S}$, we have a diffeomorphism $\mathcal{D}$ written as

$$\mathcal{D}_{(\mathbb{S})} : \mathrm{A} \to \mathcal{D}_{(\mathbb{S})}(\mathrm{A}) = \mathrm{A}^2 + \partial_\tau \mathrm{A} \int \mathrm{A} d\tau. \quad (46)$$

[7] The generators of supersymmetry are elements of the adS Clifford Algebra $\mathbb{C}_{3,2}$ and, at the same time, elements of the orthogonal group $\mathbb{O}(3,2)$ that represent Wick-rotations when acting on gravitors. The reasons by which I applied an adS Clifford algebra for supergravity with gravitorial affinors is that 1) Clifford algebras usually furnishes spinorial representations of rotation groups and 2) supergravity does not exist without anti-de Sitter space [30].

It is interesting to make

$$\mathrm{A}' = \mathrm{A}^2 + \partial_\tau \mathrm{A} \int \mathrm{A} d\tau = \mathrm{A}^2 - \frac{1}{\alpha^2}(\partial_\mu \mathrm{A})^2 = \tag{47}$$

$$= (\mathrm{A} - \frac{1}{\alpha}\partial_\mu \mathrm{A}).(\mathrm{A} + \frac{1}{\alpha}\partial_\mu \mathrm{A}), \tag{48}$$

where α is a constant. In fact, this transformation maps an object into another whose locality is arrested from far away in time. Since we are dealing with a diffeomorphism $\mathcal{D} : \mathrm{A} \to \mathcal{D}(\mathrm{A})$, as the map $\mathcal{D}$ is invertible, we expect to find a function A that preserves the invariance of the Lagrangian. From the above imposition, we get a simple integro-differential equation

$$-\frac{1}{\alpha^2}(\partial_\mu \mathrm{A})^2 = \partial_\tau \mathrm{A} \int \mathrm{A} d\tau. \tag{49}$$

The left-hand side is the spacelike (local) remainder of field evolution, while the right-hand side is the instantaneous field status under influence of the field history (non-local)[8]. Taking one spatial dimension solely, a solution is

$$\mathrm{A} = A e^{i(\alpha\mu + \beta\tau)} \begin{pmatrix} \boldsymbol{\imath}_2 \\ \sigma_3 \end{pmatrix}, \tag{50}$$

where the column object is one gravitorial representation of the gravitino in adS Clifford algebra. This solution is nothing more than the «shadow» gravitational wave associated to gravitino's polarization. Returning to my first Lagrangian, if we assume (45) as a universal supersymmetric transformation for gravity, any field A shall behave in this way. Them, after the appropriate substitutions,

$$\mathcal{L} = M^2 \mathrm{A}\mathrm{A}' \partial_\tau \mathrm{A}' \int \mathrm{A} d\tau + 1/3 M^2 \mathrm{A}'^3 + i\breve{r}\partial_\tau \breve{r} =$$

$$= M^2 \mathrm{A}(\mathrm{A} - 1/\alpha\partial_\mu \mathrm{A}).(\mathrm{A} + 1/\alpha\partial_\mu \mathrm{A})\partial_\tau \left[(\mathrm{A} - 1/\alpha\partial_\mu \mathrm{A}).(\mathrm{A} + 1/\alpha\partial_\mu \mathrm{A})\right].$$

$$.\int \mathrm{A} d\tau + 1/3 M^2 \left[(\mathrm{A} - 1/\alpha\partial_\mu \mathrm{A}).(\mathrm{A} + 1/\alpha\partial_\mu \mathrm{A})\right]^3 + i\breve{r}\partial_\tau \breve{r}. \tag{51}$$

Calculations lead to confirm Lagrangian invariance

$$\mathcal{L} = \frac{7}{3} M^2 A^6 e^{6i(\alpha\mu + \beta\tau)} \begin{pmatrix} \boldsymbol{\imath}_2 \\ \sigma_3 \end{pmatrix}^6 + i\breve{r}\partial_\tau \breve{r}, \tag{52}$$

[8] This equality aims to ensure that the local inhomogeneity in space has roots in the remote past.

$$\mathcal{L}' = \frac{32}{3} M^2 A^6 e^{6i(\alpha\mu+\beta\tau)} \begin{pmatrix} \mathbb{i}_2 \\ \sigma_3 \end{pmatrix}^6 + i\breve{r}\partial_\tau \breve{r}. \tag{53}$$

This shows that, at first, the presence of non-local terms do not affects Lagrangian symmetry properties. As the amplitude A has in general no dimension, de first term is rigorously a mass term and the mass term difference observed between $\mathcal{L}$ and $\mathcal{L}'$ is said a diffeomorphic mass difference. The column term

$$\begin{pmatrix} \mathbb{i}_2 \\ \sigma_3 \end{pmatrix}^6$$

is such that

$$\begin{pmatrix} \mathbb{i}_2 \\ \sigma_3 \end{pmatrix}^6 = \begin{pmatrix} \mathbb{i}_2 \\ \sigma_3 \end{pmatrix}^2 = \begin{pmatrix} -\mathbb{1}_2 \\ \mathbb{1}_2 \end{pmatrix}. \tag{54}$$

In addition, to understand the role of field $\breve{r}$, never is overmuch to remind the content of Noether's theorem. For a system with Lagrangian density of type $\mathcal{L} = \mathcal{L}(\Phi; \dot{\Phi}, \boldsymbol{\nabla}\Phi)$, a continuous symmetry of $\mathcal{L}$ generates an equation of continuity $\frac{\partial}{\partial\tau}\rho + \boldsymbol{\nabla}.\mathbf{j} = 0$, where ρ and $\mathbf{j}$ are functionals of $\Phi, \dot{\Phi}, \boldsymbol{\nabla}\Phi$, so that $Q = \int d^3\mathbf{x}\rho(\Phi; \dot{\Phi}, \boldsymbol{\nabla}\Phi)$ is a constant of motion. As pointed out by O'Raifeartaigh, «The Noether theorem gives the general relationship between symmetries and conservation laws. [...] Thus to every symmetry there corresponds a conserved quantity and conversely » [28]. So, from Noether's theorem applied to Lagrangian (36), considering a transformation that requires only a displacement in time, there is a conservation expression on the Hamiltonian

$$\frac{d}{d\tau}\left(\frac{\partial\mathcal{L}}{\partial\partial_\tau\langle\breve{G}\rangle}\partial_\tau\langle\breve{G}\rangle - \mathcal{L}\right) = 0. \tag{55}$$

Thereby, from $\partial_\tau\langle\breve{G}\rangle$ we get

$$\frac{d}{d\tau}\left\{-1/3M^2\langle\breve{G}\rangle^3 - i\breve{r}\partial_\tau\breve{r}\right\} = 0. \tag{56}$$

Let us imagine, for simplicity, that the current term is negligible (the amplitude of the current is very small) with respect to the self-interaction mass term. So,

$$i\breve{r}\partial_\tau\breve{r} = -\frac{1}{3}M^2\langle\breve{G}\rangle^3. \tag{57}$$

The field $\breve{r}$ is called «junction field» or «filtrino», because it is defined at the junction of spacetimes adS and dS, and because it seems to «filter» the mass of gravitino when it collides with the internal side of the junction.

Now, the reader must understand that kets $\langle\ \rangle$ are applied at the junction (symbolizing interaction both on and off the edge), while kets $|\ \rangle$ are referring to actions coming from the inside out the junction, and kets $\langle\ |$ related to actions coming from outside inward the junction. Assuming expression (50), field $\langle\breve{G}\rangle$ gains the form

$$\langle\breve{G}\rangle = A^2 e^{i2(\alpha\mu+\beta\tau)}\begin{pmatrix}\mathbb{i}_2\\ \sigma_3\end{pmatrix}^2 + Ai\beta e^{i(\alpha\mu+\beta\tau)}\frac{A}{i\beta}e^{i(\alpha\mu+\beta\tau)}\begin{pmatrix}\mathbb{i}_2\\ \sigma_3\end{pmatrix}^2 =$$

$$= 2A^2 e^{i2(\alpha\mu+\beta\tau)}\begin{pmatrix}\mathbb{i}_2\\ \sigma_3\end{pmatrix}^2. \tag{58}$$

Accordingly,

$$i\breve{r}\partial_\tau\breve{r} = -\frac{1}{3}M^2\langle\breve{G}\rangle^3 = -\frac{1}{3}M^2 8A^6 e^{i6(\alpha\mu+\beta\tau)}\begin{pmatrix}\mathbb{i}_2\\ \sigma_3\end{pmatrix}^6. \tag{59}$$

The integration gives

$$\breve{r}^2 = -\frac{16}{3i}M^2\frac{A^6}{6\beta i}e^{i6(\alpha\mu+\beta\tau)}\begin{pmatrix}\mathbb{i}_2\\ \sigma_3\end{pmatrix}^6 = \frac{8}{9}\frac{M^2A^6}{\beta}e^{i6(\alpha\mu+\beta\tau)}\begin{pmatrix}\mathbb{i}_2\\ \sigma_3\end{pmatrix}^6; \tag{60}$$

$$\breve{r} = \frac{1}{3}\sqrt{\frac{8}{\beta}}MA^3 e^{i3(\alpha\mu+\beta\tau)}\begin{pmatrix}\mathbb{i}_2\\ \sigma_3\end{pmatrix}^3; \tag{61}$$

$$\breve{r} = \frac{1}{3}\sqrt{\frac{8}{\beta}}MA^3 e^{i3(\alpha\mu+\beta\tau)}\begin{pmatrix}-\mathbb{i}_2\\ \sigma_3\end{pmatrix}. \tag{62}$$

We may note that $\begin{pmatrix}-\mathbb{i}_2\\ \sigma_3\end{pmatrix}$ is in fact a Wick-rotation[9] of a graviton gravitorial representation given by

$$-\begin{pmatrix} 0 & \begin{vmatrix}1 & 0\\ 0 & -1\end{vmatrix}\\ \begin{vmatrix}-1 & 0\\ 0 & 1\end{vmatrix} & 0\end{pmatrix}\begin{pmatrix}\mathbb{1}_2\\ i\sigma_3\end{pmatrix} = \begin{pmatrix}-\mathbb{i}_2\\ \sigma_3\end{pmatrix}, \tag{63}$$

[9] Wick-rotations were applied in my theory after Nieuwenhuizen and Waldron[27], which have done the proposal of «a continuous Wick-rotation for Dirac, Majorana and Weyl spinors from Minkowski spacetime to Euclidean space, which treats fermions on the same footing as bosons», emphasizing that the study focuses the Wick-rotation of the field theory itself.

where the matrix $\vartheta_3 = \begin{pmatrix} 0 & \begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} \\ \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} & 0 \end{pmatrix}$ lives in adS Clifford algebra $\mathbb{C}_{3,2}$.

So,

$$\breve{r} = -\frac{1}{3}\sqrt{\frac{8}{\beta}} M A^3 e^{i3(\alpha\mu+\beta\tau)} \vartheta_3 \begin{pmatrix} \mathbb{1}_2 \\ i\sigma_3 \end{pmatrix}. \tag{64}$$

In short, I conclude that the proposed paleogravity Lagrangian density provides:

1. an interpretation of the mediator field $\langle\breve{G}\rangle$ as a quadratic transformation of the field $|g\rangle$; if this rule of transformation is applied to any field of the form (50), Lagrangian symmetry is preserved.
2. an auxiliary field $\breve{r}$ to describe the mass-interchange mechanism at the boundary of the G-closure, which is internally adS and dominated by gravitinos.

As can be seen, paleogravity is not a quantum representation but a meta-framework created on symmetries capable of producing a non-local image of gravity, described by classical fields easily linkable to general relativity. Perhaps gravity is never made a quantum theory in terms of elementary particles, being gravitons and gravitinos only names that symbolize geometrodynamical symmetries . It was precisely this belief that led me to build a quantum spacetime model as shall be seen below.

Exercise 2.11 *Given the lagrangian,*

$$\mathcal{L} = M^2|g\rangle\langle\breve{G}\rangle\partial_\tau\langle\breve{G}\rangle \int |g\rangle d\tau + 1/3M^2\langle\breve{G}\rangle^3 + i\breve{r}\partial_\tau\breve{r},$$

and taking the coordinate-field $\int |g\rangle d\tau$, *find an expression for* $\langle\breve{G}\rangle$, *proving that it is satisfied for*

$$\langle\breve{G}\rangle = A^2 e^{i2(\alpha\mu+\beta\tau)} \begin{pmatrix} \boldsymbol{i}_2 \\ \sigma_3 \end{pmatrix}^2 + Ai\beta e^{i(\alpha\mu+\beta\tau)} \frac{A}{i\beta} e^{i(\alpha\mu+\beta\tau)} \begin{pmatrix} \boldsymbol{i}_2 \\ \sigma_3 \end{pmatrix}^2 =$$

$$= 2A^2 e^{i2(\alpha\mu+\beta\tau)} \begin{pmatrix} \boldsymbol{i}_2 \\ \sigma_3 \end{pmatrix}^2. \tag{65}$$

Hint: *apply Euler equation for* $\int |g\rangle d\tau$.

2.2 Gravity as a gauge theory

In the context of GR, if we think about rigid motions in spacetime, we see that these motions are in fact gauge transformations, as it can be confirmed by the parallel transport of a vector given from Christoffel connection, say

$$dv^{\alpha} = -v^{\mu} \Gamma^{\alpha}_{\mu\sigma} dv^{\sigma} \tag{66}$$

or

$$dv_{\mu} = v_{\alpha} \Gamma^{\alpha}_{\mu\sigma} dv^{\sigma}. \tag{67}$$

From here, once that the scalar product of two vectors at the same point is

$$u.v = g_{\mu\nu} u^{\mu} v^{\nu} \therefore \tag{68}$$

$$v.v = g_{\mu\nu} v^{\mu} v^{\nu} = |v|^2 = (v_{\nu}, v^{\nu}) \ \text{(squared length)}, \tag{69}$$

it is simple to verify that the length of a vector is invariant under parallel transport, that is,

$$\begin{aligned} d\,|v|^2 &= d\,(v_{\nu}, v^{\nu}) \\ &= dv_{\nu} v^{\nu} + v_{\nu} dv^{\nu} \\ &= v_{\alpha} \Gamma^{\alpha}_{\mu\sigma} dv^{\mu} v^{\sigma} - v_{\nu} v^{\mu} \Gamma^{\nu}_{\mu\alpha} dv^{\alpha} = 0. \end{aligned}$$

Now, paying attention to some notation adjustments, we can express global spacetime transformations as

$$x'^{\mu} = \chi^{\mu}_{\nu} x^{\nu} + a^{\mu} \ \text{(corresponding to Lorenz plus translations)}. \tag{70}$$

Accordingly previous explanation, local implementation, however, requires at each point of spacetime

$$x'^{\mu} = \chi^{\mu}_{\nu}(x) x^{\nu} + a^{\mu}(x), \tag{71}$$

that is,

$$dx'^{\mu} = \chi^{\mu}_{\nu}(x) dx^{\nu}. \tag{72}$$

The invariance of the geodesic arc element (or the coordinate invariance of derivatives) is gained by the introduction of a new metric tensor field

$$ds^2 = g_{\mu\nu}(x) dx^{\mu} dx^{\nu} = g'_{\mu\nu}(x) dx'^{\mu} dx'^{\nu}, \tag{73}$$

which transforms in accordance to

$$g'_{\gamma\eta}(x') = \chi^{-1\mu}_{\gamma}(x) g_{\mu\nu}(x) \chi^{-1\nu}_{\eta}(x). \tag{74}$$

To preserve the homogeneity of tensor transformations, the covariant derivative must obey

$$D_\mu A^\alpha \rightarrow D'_\mu A^\alpha = \chi^\nu_\mu \chi^\alpha_\beta D_\nu A^\beta. \tag{75}$$

This condition may be achieved by

$$D_\mu A^\alpha = \partial_\mu A^\alpha + \Gamma^\alpha_{\mu\sigma} A^\sigma, \tag{76}$$

with the connection

$$\Gamma^\alpha_{\mu\sigma} = \frac{1}{2} g^{\alpha\rho} \left[\partial_\mu g_{\sigma\rho} + \partial_\sigma g_{\mu\rho} - \partial_\rho g_{\mu\sigma}\right]. \tag{77}$$

Despite all this, the gauge of gravitation is entirely based on the geometric concept of the gravitational field, which derives directly from the spacetime structure, unlike the other physical fields.

Exercise 2.21 *Prove that for a change of coordinates, from primed to unprimed, the simple partial derivative yields to a non-homogeneous tensorial transformation.*

2.3 Teleparalell gravity in its fundamentals

Within the context of gauge theories, it should be mentioned the teleparallel gravity relating to the translation group. In this theory, each point of spacetime carries an associated Minkowski tangent space over which the translation group — the gauge group as such — acts. The crucial gauge field is the translational potential B^a_μ which takes values in the corresponding Lie algebra $\mathrm{B}_\mu = \mathrm{B}^a_\mu \partial_a$, where ∂_a is the generator of the infinitesimal translations. The anholonomic indice a comes from the Minkowski metric assumed, that is, $\eta_{ab} = (+1, -1, -1, -1)$. The most important feature of this model is the introduction of a *vierbein* (or tetrad) field that can be applied to define the linear Weitzenböck connection . This connection presents torsion, not curvature. In teleparallelism, torsion accounts for gravity in a mechanistic way, going in the opposite direction to that of GR.

Under a local infinitesimal translation of the tangent space coordinates, say ϵ^a, the gauge field trasnforms as

$$\mathrm{B'}^a_\mu = \mathrm{B}^a_\mu - \partial_\mu \epsilon^a. \tag{78}$$

Vierbein field h^a_μ then rises according to

$$h^a_\mu = \partial_\mu x^a + \mathrm{B}^a_\mu. \tag{79}$$

Given the above tetrad, which represents four linearly independent fields built by the mapping from tangent space to Minkowski space, we may write the Weitzenböck torsion connection

$$\Gamma^{\rho(W)}_{\mu\nu} = h_a^\rho \partial_\nu h_\mu^a, \tag{80}$$

with no curvature. Torsion tensor is written as

$$T^\rho_{\nu\mu} = \Gamma^{\rho(W)}_{\mu\nu} - \Gamma^{\rho(W)}_{\nu\mu} = h_a^\rho \left(\partial_\nu h_\mu^a - \partial_\mu h_\nu^a\right), \tag{81}$$

from which we get the field strength (the torsion written in the tetrad basis)

$$F^a_{\nu\mu} = h^a_\rho T^\rho_{\nu\mu}. \tag{82}$$

Then, torsion is the field strength of the translational group . Physicists have studied teleparallel gravity with greater interest from the beginning of this century with eyes towards quantization of the fourth interaction, but non-zero torsion-based gravity has several open issues; for instance, black-holes are found to have different behavior according to curvature and torsion analysis. My opinion, which in fact is not only mine, is that in the current state of knowledge in which we are, to take a model of gravity based on torsion or curvature is merely a matter of personal preferences. Particularly, the introduction of connections with torsion and no curvature seems to be inappropriate to modeling spacetime evolution from timelike or spacelike paths, since in the teleparallel equivalent of GR there are no geodesics at all, but force equations. Although this might seem very attractive, there is something of a throwback to the old mechanical design, especially the classical idea of force. I believe that we are not looking for a nostalgic view but a really new physics.

2.4 The gauge in Lyra's geometry

Many works appeared on cosmology with Lyra's geometry from authors as Reddy and Innaiah [33], Reddy and Venkateswarlu [34], both in the eighties, and more recently Shchigolev [47]. Shchigolev even says that «[...] Lyra's geometry can be considered as the candidate for modification of the contemporary cosmological models, the necessity of which is almost generally recognized» [47]. As Lyra himself said [22], «[...]*Es besteht eine so nahe innere Verwandtschaft des hier gegebenen Aufbaus der Infinitesimalgeometrie mit demjenigen Weyls aus dem Jahre 1918, daß man ebensogut von einer Modifikation der Weylschen Geometrie sprechen könnte*» (There is such a close inner relationship of the infinitesimal structure given here with that from Weyl (1918) that one could just speak of a modification of Weyl's geometry)[10]. Thus, Lyra's geometry is a generalization of Riemannian ge-

[10] Translated from German by the author.

ometry[11] — initially taken in a manifold not endowed of a metric — with a positive definite function, the scalar field $\chi(x^k)$ for scale changes, in which the reference system is defined not only by the coordinates but also by including that scalar field, that is, the gauge function $\chi(x^k)$ [22][12], so that the Levi-Civita-Christofell connection is χ^{-1}-gauged and added of a negative term referring to the vector displacement of a given parallel transport between two neighboring points. Therefore, a change in reference system is in fact a change of coordinates and a gauge transformation, all at once.

A tensor metric $g_{\mu\sigma}$ is subsequently introduced, and the new asymmetric connection is given by

$$^{\dagger}\Gamma^{\alpha}_{\mu\sigma} = \chi^{-1}\Gamma^{\alpha}_{\mu\sigma} - \frac{1}{2}\left(\delta^{\alpha}_{\mu}\phi_{\sigma} + \delta^{\alpha}_{\sigma}\phi_{\mu} - g_{\mu\sigma}\phi^{\alpha}\right), \tag{83}$$

where $^{\dagger}\Gamma^{\alpha}_{\mu\sigma}$ is symmetric in only the lower indices, $\Gamma^{\alpha}_{\mu\sigma}$ is the usual connection [13], and ϕ_{σ} is the displacement vector field. The geodetic arc element in Lyra's manifold has the form

$$ds^2 = \chi^2 g_{\mu\sigma} dx^{\mu} dx^{\sigma}, \tag{84}$$

and the change from a reference frame $\left(\chi, x^i\right)$ to $\left(\chi', x'^i\right)$ is obtained doing

$$\chi' = \chi\left(\chi, x^k\right),\; x'^i = x^i\left(x^k\right). \tag{85}$$

It is important to add that the Jacobian obeys

$$\left|\frac{\partial x'^i}{\partial x^k}\right| \neq 0$$

[11] The reader can expand their skills in Riemannian geometry, for example, with reference [42].

[12] In words from Lyra: «[...]*Dabei wird der Eichbegriff nicht mehr als Festlegung von Längeneinheiten verstanden, sondern schon im strukturlosen Raum als ein mit dem Koordinatensystem gleichberechtigter Bestandteil des Bezugssystems eingeführt*» (Here, the calibration term is no longer understood as establishing length units, but introduced already in the structureless space on an equal footing with the coordinate system part of the reference system). Translated from German by the author.

[13] Whenever possible, it is desirable to reflect upon the precise meaning of the objects under study. Weaving formal considerations on the structure of Riemannian manifolds, Weitzenböck [53] summarized his conclusions by saying the following : «[...]*die Functionen $\Gamma^{\rho}_{\mu\nu}$ definieren die "infinitesimale Parallelverschiebung" der Vektoren (und damit auch die von Tensoren höherer Stufe), oder auch: die Funktionen $\Gamma^{\rho}_{\mu\nu}$ definieren den "affinen Zusammenhang" der Mannigfaltigkeit*» ([...] function $\Gamma^{\rho}_{\mu\nu}$ defines the "infinitesimal parallel displacement" of the vectors (and thus also of tensors of higher order), or else function $\Gamma^{\rho}_{\mu\nu}$ defines the "affine relation" of the manifoldness). Translated from German by the author. Thus, Weitzenböck understands function $\Gamma^{\rho}_{\mu\nu}$ as the analytical representation of the structural geometrical essence of a Riemannian manifold, ultimately its "holonomyness" rephrased in operational description encoded by an algorithm of parallel transport .

and

$$\frac{\partial \chi'}{\partial \chi} \neq 0.$$

Lyra's geometry has the fundamental property that the length of a vector in parallel transport does not change, in contrast with Weyl's geometry.

Gauge theories give a unique possibility of describing, in the framework of quantum field theory, the phenomenon of asymptotic freedom (Faddeev and Slavnov, 1980).

2.5 The quantum spacetime in Lyra's geometry ✓

Recently, focusing some exotic effects in the interaction of two supermassive bodies, I proposed a new approach on quantum gravity in which it is considered — having in mind that any region in space is continually being expanded (or compressed), so that there are no rigid structures at all[14] — a metric in singularity functions , making it possible to analyze any infinitesimal timelike element of a geodesic in a gravitational singularity with no vanishing of space components of the metric tensor, but nulling the participation of space in the geodesic path simply choosing a value of the spacelike x-variable for the infinitesimal element in Macauley kets , $d\left\langle x-\varepsilon\right\rangle_{\alpha}^{2}$, with the restriction $x<\varepsilon_{\alpha}$ [46]. Accordingly, the space still exists in the singularity, however, as it was «frozen». This means that the geometry of spacetime fluctuates (or undergoes excitations) over «non-space», apart from the trivial case of the $g_{\mu\sigma}=0$ solution [44]. Such a work refers to a phenomenological theory concerning a possible effect of time machine between two massive bodies interacting with one another. Unfortunately, current criticism on physics often lacks of considerations on conceptual and semantic structures. Once the work was based on a proposition about the behavior of a black-hole binary system, I would like to clarify the term «proposition», since proposition is only a sentence that can be true or false, a statement to be proved, explained, or discussed [15]. In the referred work, it is about a lawlike statement depending obviously of further observational corroboration. Either in math or physics, the meaning of «proposition» is basically the same, differing only in the essence of the verification process. From Bunge's analysis of specific lawlike statements (LLS), I briefly conclude that the proposition enunciated in reference [46] is

[14] In fact, it is quite comfortable to take on this premise, even if one consider simple thought experiments in special relativity, since in a perfectly rigid object the speed of sound would be infinite, contradicting the principle that the highest speed is the speed of light.

[15] This discussion with eminent colleagues physicists from Bulgaria was particularly important for advancing the work in question.

1. **Regarding their referents** — property-referent LLS: proposition referring to constant relations among selected aspects of facts or properties of entities.
2. **Regarding its precision** — predicate-imprecise LLS: proposition containing coarse predicates, like «strong», which lack extensional and/or intensional precision.
3. **Regarding its structure of predicates** — existential LLS: limited scope proposition involving two or more atomic predicates.

There are several other items of propositional classification in Bunge's work , but these three seem sufficient (the reader must acquaint himself with that proposition in the given reference). The phenomenological model presented at Planck scale brings the advantage of establishing some reasonable physical predictions about the spacetime behavior under the intense gravitational compression of two supermassive bodies, and introduces an original way to match quantum spacetime with quantum Riemannian metric in accordance with Einstein's field equations . I wish I could present a more extensive discussion, confronting various theories. However, even if there was space in these notes, this would be an impossible task, since the necessary availability for that is beyond my possibilities at the moment. Therefore, I want to emphasize only my investigations to make compatible with GR the Planckian dimensions of certain gravitational singularities where the shortdistance quantum nature of spacetime becomes relevant.

From the above scenario, since no effective displacement occurs, field becomes static in space, so that the connection

$$\Gamma^{\alpha}_{00} = \frac{1}{2} g^{\alpha\rho} \left(\partial_0 g_{\rho 0} + \partial_0 g_{0\rho} - \partial_\rho g_{00} \right) \tag{86}$$

reduces to

$$\Gamma^{\alpha}_{00} = \frac{1}{2} g^{\alpha\rho} \left(\partial_0 g_{\rho 0} + \partial_0 g_{0\rho} \right), \tag{87}$$

in which

$$\partial_0 g_{\rho 0} = \frac{\partial g_{\rho 0}}{\partial \langle t - \varepsilon_0 \rangle} = \frac{\partial g_{\rho 0}}{\partial \langle x - \varepsilon \rangle_0};$$

$$\partial_\rho g_{00} = \frac{\partial g_{00}}{\partial \langle x - \varepsilon_\rho \rangle} = \frac{\partial g_{00}}{\partial \langle x - \varepsilon \rangle_\rho}.$$

The quantum spacetime was matched with quantum Riemannian metric in order to obtain the correlation function

$$\langle 0 | \, g_{\mu\sigma} d\langle x - \varepsilon \rangle_\mu d\langle x - \varepsilon \rangle_\sigma \, | 0 \rangle = -d\langle x - \varepsilon \rangle_0^2. \tag{88}$$

Although it has been produced a certain number of works applying Lyra's geometry , very little effectively was gained so far, except, perhaps, the interpretation related to the cosmological constant as I shall discuss below. Nevertheless, the search for a suitable physics to describe gravitational singularities led me to a complex geometry resulting from a combination of Lyra's geometry with the geometry of singularity functions described in [44]. Using Lyra's geometry, the gauged connection gets the general form (83), which restricted to timelike singularity coordinates gives

$$^{\dagger}\Gamma^{\alpha}_{00} = \frac{1}{2}\chi^{-1}g^{\alpha\rho}\left(\partial_0 g_{\rho 0} + \partial_0 g_{0\rho}\right) - \frac{1}{2}\left(\delta^{\alpha}_{0}\phi_0 + \delta^{\alpha}_{0}\phi_0\right), \tag{89}$$

with

$$\phi^0 = \phi_0 = \beta_{\langle t-\varepsilon_0\rangle}{}^{16} \tag{90}$$

$$\phi^{\alpha} = \phi_{\alpha} = \phi_{\langle x-\varepsilon\rangle_{\alpha}}, \tag{91}$$

and

$$\phi_{\langle x-\varepsilon\rangle_{\alpha}} = 0$$

for $x < \varepsilon_{\alpha}$.

The argument that the indices simplify the formalism is really a scam. Therefore, unlike the literature in general, we shall make a careful explanation of the meaning of these expressions. As stated, $\Gamma^{\alpha}_{\mu\sigma}$ is symmetric only in lower indices, which means that "α" does not commute, in general, with indices "μ" and "σ", appearing as superscript symbolizing contravariance, i.e., infinitesimal displacement. Also, in accordance with previous deductions that led to the geodesic equation in singularity functions [44], indices "μ" and "σ" were taken as time-labels while "α" and "ρ" became space-labels ("ρ" replaces "α" to characterize the metric tensor component as a function of time and space in partial derivatives, but this is done without any loss of generality). Thus, according to the second term in the right-hand side of expression (89), those infinitesimal displacements run over time, on the temporal component of the vector field, in the spatial directions "α" of this field. However, as there is no spatial displacement (see properties of singularity functions, taking care not to confuse "spatial direction" with "spatial displacement"), the spatial components of the displacement vector field cancel out, thus leaving the expression (89).

[16] In my previous work [46], the adoption of singularity functions aimed to allow disregard the participation of space in the calculation of the invariant commoving timelike element, with no need to guess lack of space. Thus, timelike geodesics are determined by application of the properties of Macauley kets on their space parts, since the usual differential coordinates were replaced by differentials of intervals. Thus,

$$\phi_{\mu} = \left(\beta_{\langle t-\varepsilon\rangle_0}, \langle x-\varepsilon\rangle_1, \langle x-\varepsilon\rangle_2, \langle x-\varepsilon\rangle_3\right) \rightarrow (\beta, 0, 0, 0)$$

for $x_{\mu} < \varepsilon_{\mu}$.

The parallel transfer of a vector ω in Lyra's geometry is given by

$$\delta\omega^{\alpha} = -\left({}^{\dagger}\Gamma^{\alpha}_{\mu\sigma} - \frac{1}{2}\delta^{\alpha}_{\mu}\phi_{\sigma}\right)\omega^{\mu}\chi dx^{\sigma}. \tag{92}$$

If one assumes the natural gauge ($\chi = 1$), the vector length is not changed under parallel transfer.

In its general form, the geodesic is now described by

$$\chi\frac{d^2\langle x-\varepsilon\rangle_{\alpha}}{d\tau^2} + {}^{\dagger}\Gamma^{\alpha}_{\mu\sigma}\frac{\chi d\langle x-\varepsilon\rangle_{\mu}}{d\tau}\frac{\chi d\langle x-\varepsilon\rangle_{\sigma}}{d\tau} = 0; \tag{93}$$

$$\frac{d^2\langle x-\varepsilon\rangle_{\alpha}}{d\tau^2} + \left[\chi^{-1}\Gamma^{\alpha}_{\mu\sigma} - \frac{1}{2}\left(\delta^{\alpha}_{\mu}\phi_{\sigma} + \delta^{\alpha}_{\sigma}\phi_{\mu} - g_{\mu\sigma}\phi^{\alpha}\right)\right] \times\frac{d\langle x-\varepsilon\rangle_{\mu}}{d\tau}\chi\frac{d\langle x-\varepsilon\rangle_{\sigma}}{d\tau} = 0; \tag{94}$$

$$\frac{d^2\langle x-\varepsilon\rangle_{\alpha}}{d\tau^2} + \Gamma^{\alpha}_{\mu\sigma}\frac{d\langle x-\varepsilon\rangle_{\mu}}{d\tau}\frac{d\langle x-\varepsilon\rangle_{\sigma}}{d\tau} -\frac{\chi}{2}\left(\delta^{\alpha}_{\mu}\phi_{\sigma} + \delta^{\alpha}_{\sigma}\phi_{\mu} - g_{\mu\sigma}\phi^{\alpha}\right)\frac{d\langle x-\varepsilon\rangle_{\mu}}{d\tau}\frac{d\langle x-\varepsilon\rangle_{\sigma}}{d\tau} = 0. \tag{95}$$

Lastly, for timelike geodesics in singularity representation,

$$\frac{d^2\langle x-\varepsilon\rangle_{\alpha}}{d\tau^2} + \Gamma^{\alpha}_{00}\frac{d\langle x-\varepsilon\rangle_{0}}{d\tau}\frac{d\langle x-\varepsilon\rangle_{0}}{d\tau} - \frac{\chi}{2}\left(\delta^{\alpha}_{0}\phi_{0} + \delta^{\alpha}_{0}\phi_{0}\right)\frac{d\langle x-\varepsilon\rangle_{0}}{d\tau}\frac{d\langle x-\varepsilon\rangle_{0}}{d\tau} = 0. \tag{96}$$

An obvious advantage of Lyra's geometry is that under the new connection a vector length is unchanged after a parallel transfer, which is physically appropriate, especially in the case of displacements only in time, whose comprehension is far from trivial. Also, as yet we'll see below, Lyra's geometry has raised new interpretations to the cosmological constant from Einstein's equations .

We consider, for instance, the FLRW background. As we know, Einstein's field equation in Lyra geometry is

$$G_{\mu\sigma} + \frac{3}{2}\phi^{\mu}\phi_{\sigma} - \frac{3}{4}\delta_{\mu\sigma}\phi^{\alpha}\phi_{\alpha} = -\kappa T_{\mu\sigma}, \tag{97}$$

which gives

$$G_{00} + \frac{3}{2}\phi^{0}\phi_{0} - \frac{3}{4}\delta_{00}\phi^{0}\phi_{0} = -\kappa T_{00}; \tag{98}$$

$$G_{11} + \frac{3}{2}\phi^{1}\phi_{1} - \frac{3}{4}\delta_{11}\phi^{1}\phi_{1} = -\kappa T_{11}; \tag{99}$$

$$G_{22} + \frac{3}{2}\phi^2\phi_2 - \frac{3}{4}\delta_{22}\phi^2\phi_2 = -\kappa T_{22}; \tag{100}$$

$$G_{33} + \frac{3}{2}\phi^3\phi_3 - \frac{3}{4}\delta_{33}\phi^3\phi_3 = -\kappa T_{33}. \tag{101}$$

Restricted to timelike geodesics, as in the quantum theory of spacetime presented in [46], we stay with

$$G_{00} + \frac{3}{2}\phi^0\phi_0 - \frac{3}{4}\delta_{00}\phi^0\phi_0 = -\kappa T_{00}. \tag{102}$$

From this, the Friedmann-like field equation is written as

$$G_{00} + \frac{3}{4}\delta_{00}\beta^2 = -\kappa T_{00}, \tag{103}$$

or

$$-G_{00} - \frac{3}{4}\delta_{00}\beta^2 = \kappa T_{00}. \tag{104}$$

Since we have

$$-G_{00} = \frac{3k}{R^2} + 3\left(\frac{\dot{R}}{R}\right)^2, \tag{105}$$

them

$$3\left(\frac{\dot{R}}{R}\right)^2 + \frac{3k}{R^2} - \frac{3}{4}\beta^2_{\langle t-\varepsilon\rangle_0} = \kappa\rho_{\langle t-\varepsilon\rangle_0},$$

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} - \frac{1}{4}\beta^2_{\langle t-\varepsilon\rangle_0} = \frac{8\pi G}{3}\rho_{\langle t-\varepsilon\rangle_0}. \tag{106}$$

All the letters designate the well known quantities of GR and cosmology, unless otherwise indicated. Nowadays, many authors understand the constant displacement vector field in Lyra formalism with the same physical role as the cosmological constant in the standard GR. In this sense, we can say that the cosmological constant naturally results from the introduction of the Lyra gauge. Therefore, it is expected the new gauge could reflect the characteristics of the cosmological constant term, that is

$$\phi^1\phi_1 = \phi^2\phi_2 = \phi^3\phi_3. \tag{107}$$

The meaning of the parallel transfer of a time interval

Timelike geodesics are treated with a bit of common sense even if one understands that it is in a conceptual level very far from a naive physical framework. This is so because time in GR is not the time of clocks but an evolutionary variable, and it is difficult to us to discard old archetypes like rules and clocks.

Whenever we seek a new physics to give account for an almost impenetrable phenomenon we try to find the invariants of the theory, the referents that make possible to get some knowledge about, and this search unwittingly drags us again to the classical measuring tools for thought experiments. From my point of view, the most interesting thing about the introduction of Lyra's gauge is the feasibility of the description of a notional parallel transfer in time without changing the duration, regardless of the spatial direction. This is an invariant useful to describe one of the quantum faces of gravity.

As we have seen briefly, physicists try to interpret the real meaning of Lyra's extra-displacement terms in Einstein's equations giving to them the role of cosmological constant. Nevertheless, in my approach we have to return to Lyra's geometry discussing what is a time parallel transfer of a time interval in a certain direction. I remember that space is «frozen» in the singularity representation of a timelike geodesic; there is no space displacement. Therefore, in the natural gauge a time parallel transfer of a time interval is in fact a projection of this time interval in one space direction targeting another virtual geodesic path in which space coordinates would be also treated by singularity functions. This is a way to say that, under the same conditions, we have the same behavior of nature. In my work, these conditions feature the so-called G-closure [17]. Importantly, this geometric review in no way precludes the representation of the cosmological constant; rather, it emphasizes the invariance of the duration under parallel transport, thus characterizing a constancy of nature.

[17] In section **2.1**, I argued for a G-closure in a semiclassical approach where it was supposed the existence of gravity superpartners. Now, the situation is very different, since there are no superpartners but quanta of spacetime.

III PART: COSMOLOGY

...if the variety of things we perceive in this extraordinarily varied world could be described in a single equation, the path that we would take from that equation to the things we perceive should be terribly long and quite difficult to follow.
Hermann Bondi

3 The standard model and its scourge

From the many objections that have been made to the standard model, we shall comment only some of the most relevant. As once told Raychaudhuri, «[...] if standard cosmology were completely successful, there would hardly be any need to explore other models of the universe, except perhaps for mathematical recreation» [32][18]. Behind this observation is the fact that isotropy herewith homogeneity are accepted so to say *ad hoc*, since there is no solid empirical basis to ensure both. Tolman already warned that we should not radicalize a belief in a homogeneous universe, hinting the limitations arising from our observational condition [49]. Usually it is accepted the cosmic background radiation as an indisputable indicator of a Big-Bang and an isotropic universe in its own origin. However, speculations about the existence of strong magnetic fields in the early stages of the universe not only contradict the isotropic model, but severely affect the current conception of the meaning of the cosmic background radiation.

A great paradox emerges from the adopted metric in the standard model. Let us begin with the current ansatz

$$ds^2 = \pm dt^2 \mp \frac{R^2}{\left(1+\frac{kr^2}{4}\right)^2}\left(dr^2 + r^2 d\theta^2 + r^2\sin^2\theta d\varphi^2\right), \;\; k = 0, +1, -1, \tag{108}$$

with R (an arbitrary function of time t) obeying Einstein's field equations

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho_{(t)} \tag{109}$$

and

$$\dot{\rho}_{(t)} + 4\rho_{(t)}\frac{\dot{R}}{R} = 0 \tag{110}$$

[18] In fact, Raychaudhuri's initial motivation was restricted to a universe represented by a timeŨdependent geometry with no assumptions of homogeneity or isotropy.

for the early times, when the universe was dominated by radiation, being ρ the energy density of the matter. In such circumstances, we may put the second equation as

$$4\rho_{(t)}\dot{R} = -\dot{\rho}_{(t)}R; \tag{111}$$

$$\frac{4\dot{R}}{R} = -\frac{\dot{\rho}_{(t)}}{\rho_{(t)}}. \tag{112}$$

To solve this equation we assume exponential functions as

$$R = Me^{\gamma t}; \dot{R} = \gamma Me^{\gamma t};$$

$$\rho = Ne^{\lambda t}; \dot{\rho}_{(t)} = \lambda Ne^{\lambda t},$$

where M and N are constants. This provides

$$\frac{4\gamma Me^{\gamma t}}{Me^{\gamma t}} = -\frac{\lambda Ne^{\lambda t}}{Ne^{\lambda t}}; \tag{113}$$

$$4\gamma = -\lambda \therefore \gamma = -\frac{\lambda}{4}. \tag{114}$$

Now, we can write

$$\rho R^4 = const. = C_1. \tag{115}$$

Returning to the first Einstein's equation, we substitute last result and gain

$$\left(\dot{R}\right)^2 = \frac{8\pi}{3}\frac{C_1}{R^2}; \tag{116}$$

$$\begin{aligned} \dot{R} &= \sqrt{\frac{8\pi C_1}{3}}\frac{1}{R}; \\ RdR &= C_2 dt; \\ R^2 &= 2C_2 t; \\ R &= C_3 t^{1/2}. \end{aligned} \tag{117}$$

From the above metric, it is simple to see that for any signal we must have

$$dt^2 - \frac{R^2 dr^2}{\left(1 + \frac{kr^2}{4}\right)^2} \geq 0,$$

which implies

$$\int_0^r \frac{dr}{\left(1 + \frac{kr^2}{4}\right)} \leq \int_0^t \frac{dt}{R}.$$

With $R \sim t^{1/2}$ in the ultrarelativistic state, the integral on the right side of the inequality converges, which means that, at any time t, communication can occur only up to a finite distance, a fact that configures a horizon and features a conflict with the observed isotropy of the cosmic microwave background.

These and other questions make us wonder why there were settled so many barriers against the study of inhomogeneous cosmologies. As well wrote de Vaucouleurs, «[...] With few exceptions, modern cosmology theories are variations of homogeneous and isotropic models of general relativity. Other theories are commonly referred to as "heterodox", probably a warning for students against the heresy» [52].

3.1 Anisotropic and inhomogeneous cosmologies

In a homogeneous universe, the isotropy at a point implies isotropy in all points (being isotropy the property by which the universe looks like the same in all spatial directions, that is, all directions are equal). To avoid confusion, homogeneity and isotropy does not necessarily imply one another. Anisotropic cosmological solutions may originate from inhomogeneous models like Lemaître-Tolman-Bondi cosmology, Szekeres cosmology and Stephani cosmology, or from completely homogeneous models like Gödel's cosmology. These notes shall give emphasis on the first and third cases in order to exploring the most relevant aspects of inhomogeneous cosmologies.

Lemaître-Tolman-Bondi Cosmology

At small length scales there where observed deviations from the postulated homogeneity of the universe at large scales, a fact that imposes 1) - the need to investigate whether the accelerated cosmological expansion is real, that is, whether the acceleration is not an effect of the inhomogeneity, and 2) - the necessity to look for the length scale from which the universe becomes homogeneous, if indeed it is.

Among several inhomogeneous cosmological models, the Lemaître-Tolman-Bondi (LTB) model — the simplest and perhaps the only practicable in fact — has been applied with some interesting results as an alternative to explain the universe without cosmological constant at scales $\mathcal{O}(10)h^{-1}Mpc$ or even larger. The LTB metric under the assumption of spherical symmetry in simultaneously synchronous and commoving frame can be read as a branch of solutions of the equation,

$$ds^2 = -dt^2 + b^2(r,t)\,dr^2 + R(r,t)^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{118}$$

that describes an inhomogeneous collapse of dust or, which comes to be the same, its time reversal. These solutions are given by,

$$b^2 = \frac{R'\left(r,t\right)^2}{1+f\left(r\right)}, \tag{119}$$

where the function $f(r)$ can be thought as a spatial curvature parameter and is one of the three classical LTB arbitrary functions, and R is the angular diameter distance.

In spite of the challenges it faces and the objections faced to its major presuppositions, which one expects from a LTB model is its simultaneous and reasonable agreement with data from cosmic microwave background (CMB), from type Ia supernova, from structure formation and so forth. For example, Alnes *et al.* (2006) showed that a LTB region which reduces to an Einstein-de Sitter cosmology at a radius of $1.4Gpc$ can match both the supernova data and the location of the first acoustic peak in the CMB [2].

Lastly, the phenomenon of weak gravitational lensing wins major expression as a result of inhomogeneities. I developed, in perturbative context, a formalism for the refractive index in the LTB metric, capable to aid future measurements of the degree of inhomogeneity for different redshifts [45]. That refractive index is given by

$$\bar{n} = \frac{1}{\bar{C}}\sqrt{\frac{g_{22}}{g_{11}}}e^{-\omega(t)\int \frac{\sqrt{\varepsilon_{11}/2\varepsilon_{44}}}{R(r,t)}dr}, \tag{120}$$

where ε is a small perturbation in the metric g, $\bar{C}$ is a constant of integration, and $\omega(t)$ is a function to be determined. The physical interpretation of this equality is that the null geodesic in the representation LTB adopted here is entirely determined by the scalar function $\bar{n}$, since it includes all relevant geometric information about the deflection of the light beam. As expected, the perturbation in the metric also contributes to the refractive index, hence, for the deflection.

LTB in 5D

Following the LTB formalism, for an inhomogeneous cloud of dust, spherically symmetric, described in a five-dimensional spacetime, we would have the line element given by

$$ds^2 = \frac{R'(r,t)^2 dr^2}{1+f} + R(r,t)^2\left[d\chi^2 + \sin^2\chi\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right] - dt^2, \tag{121}$$

where $(t, r, \chi, \theta, \phi)$ are synchronous-comoving coordinates and f is the usual arbitrary function of comoving coordinate r. Coordinate χ is the fifth dimension implemented in trigonometric representation. Function $R(r, t)$ remains solution of the first independent field equation

$$\dot{R}^2 = \frac{M(r)}{R^2} + f(r). \tag{122}$$

Both arbitrary functions $f(r)$ and $M(r)$ result of the integration of field equation. Einstein's equations in five dimensions take the form $\breve{G}_{\mu\nu} = -8\pi G_5 \breve{T}_{\mu\nu}$, whose non-zero components are

$$\breve{G}_{00} = 3\frac{(-2R(r,t)\dot{R}(r,t)\dot{R}'(r,t) - 2R'(r,t)\dot{R}(r,t)^2 + R(r,t)f' + 2R'(r,t)f)}{2R(r,t)^2 R'(r,t)}, \tag{123}$$

$$\breve{G}_{11} = R'(r,t)^2 \frac{(3R(r,t)\ddot{R}(r,t) + 3\dot{R}(r,t)^2 - 3f)}{R(r,t)^2(1+f)}, \tag{124}$$

$$\begin{aligned} \breve{G}_{22} = R'(r,t)^{-1}(2R(r,t)\partial_r\ddot{R}(r,t)R'(r,t) + 2R(r,t)\dot{R}(r,t)\dot{R}'(r,t) - \\ - R(r,t)f' + R'(r,t)\dot{R}(r,t)^2 - R'(r,t)f + R(r,t)^2\ddot{R}'(r,t)), \end{aligned} \tag{125}$$

$$\breve{G}_{33} = \sin^2\chi \breve{G}_{22}, \tag{126}$$

$$\breve{G}_{44} = \sin^2\theta \breve{G}_{33}. \tag{127}$$

There is very few evidence of productive applicability of metrics with more than four dimensions connected to observational data in cosmology. From a mathematical point of view, however, it is possible to describe a LTB cavity by means of a 5D metric embedded in a 4D friedmannian background. The idea is to assume that the inhomogeneity carries in the fifth dimension information able to provide it with a symmetry such that its structure remains irreducible to FLRW unless at the junction between the FLRW background and the LTB cavity. Here it is worth to make a brief discussion of cosmological symmetries. The symmetries of spacetime, or their isometries, constitute a group for which a) the identity is an isometry, b) the inverse of an isometry is a isometry, and c) the composition of two isometries is an isometry. We define the orbit of a point p as the set of all points for which p can be moved by the action of translative isometries of space. The orbits are necessarily homogeneous, i.e., all physical quantities are the same at every point. Once an invariant manifold is a set of points mappable in themselves by the isometry group, the orbits are necessarily invariant manifolds. The freedom of translation in a given space (or transfer dimension) is generally denoted by the letter "s", being assumed s $\leq$ n, where n is the number of space dimensions. An important subgroup of the isometry group, whose dimension may be considered in each p, is the isotropy group, i.e., the group of isometries that leave p fixed (rotations). In general, the dimension number of a rotation space is represented by the letter "q" being established that $\mathrm{q} \leq 1/2\mathrm{n}(\mathrm{n}-1)$, where n is again the number of space dimensions. Thus, the dimension $\mathfrak{D}$ of the isometry group of a given space is $\mathfrak{D} = \mathrm{s} + \mathrm{q}$ (translations + rotations). In fact, continuous isometries are generated by the Lie algebra of Killing vectors. The action group is characterized by the nature of its orbit in the space in question.

For a cosmological model, due to the spacetime four-dimensionality, the possible orbital dimensionalities are $\mathrm{s} = 0, 1, 2, 3, 4$. The isometry group featuring LTB models in 4D is the $\mathcal{G}_{\mathrm{s+q}} = \mathcal{G}_3$ or $\mathcal{G}(2,1)$, isomorphic to the pseudo-orthogonal real special group $\mathrm{s}+\mathrm{q}$, $SO(2,1)$. Each LTB model is characterized by a two-dimensional surface of spherical symmetry $\mathrm{s} = 2$; all observations made anywhere on the surface are rotationally symmetrical around a privileged space direction: $\mathrm{q} = 1$; therefore, $\mathfrak{D}_{LTB4D} = 2 + 1 = 3$. However, the implementation of a fifth angular dimension corresponds to the introduction of an extra degree of translational freedom $\mathrm{p} = 1$, where $\mathfrak{D}_{LTB5D} = \mathrm{s} + \mathrm{p} + \mathrm{q} = 2 + 1 + 1 = 4$. Therefore, an LTB model in 5D, as stated above, requires a group of isometry $\lfloor\!\dashv\!\nabla\mathcal{G}_4$, isomorphic to the orthogonal singular group $\mathrm{s}+\mathrm{p}+\mathrm{q}$, $SO(2,1,1)$ corresponding to the Lie inhomogeneous algebra $\mathrm{so}(2,1,1)$.

Thus, from the above discussed, we conclude that, out of the junction, the only way to obtain a LTB 4D metric reducible to FLRW would be by an unknown mechanism of spontaneous symmetry breaking $\parallel \mathrm{Q} \parallel$, such that $\parallel \mathrm{Q} \parallel \mathrm{SO}(2,1,1) \longrightarrow \mathcal{G}_3$ [19]. This study was based on the belief that the universe evolves preserving material symmetry between homogeneous and inhomogeneous regions. That symmetry could only be broken by still unknown spontaneous mechanisms.

Stephani cosmology ✓

Another alternative to ΛCDM modeling is the so-called Stephani cosmology, with its exotic and irrotational perfect fluid driving the exact solution of Einstein's equations [20]. This cosmology and their subcases do not admit in general a barotropic equation of state, a fact that probably influenced the poor literature in the subject. An atypical and interesting work, however, came from Tupper, Marais and Helayël, where these authors show that the Stephani exact solution of Einstein's equations steered by that perfect fluid

[19] The unique situation that is physically and clearly need to find a LTB 4D metric reducible to FLRW occurs at the junctions, where the manifold has to be four-dimensional.

is compatible with an uncommon «velocity» linear k-essence [50]. A curious observation of these authors is on the 5-vector potential A_M from which the expression $A_M A^M = [A \bullet A]^{(5)}$ is assumed at the unitary gauge limit of $\left[A^{(\theta)} \bullet A^{(\theta)}\right]^{(5)}$ (with $A_M^{(\theta)}$ mapped on $A_M + \partial_M \theta$), thus not compromising gauge invariance. Howsoever, having in mind that barotropic equations of state might be too restrictive, some authors have engaged in the search for something like a «thermodynamic scheme» [7] from the energy-momentum of a perfect fluid $T^{ab} = (\rho + p)\, u^a u^b + p g^{ab}$, where ρ, p and and u^a are respectively the matter-energy density, pressure and 4-velocity. Although this approach is quite interesting, it is beyond the scope of present notes.

Stelmach and Jakacka produced a fairly comprehensive paper on non-homogeneity of the universe driven its acceleration under a Stephani cosmology [48]. However, the formalism adopted is not very friendly to our purposes, so that we shall choose the formalism presented by Hashemi *et al.* [14]. So, the metric is given by

$$ds^2 = -D^2 dt^2 + V^2 \left[dr^2 + f^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)\right], \tag{128}$$

with

$$D = \frac{1 + F^2 \left(K - R K_{,R}\right)}{1 + K F^2} \tag{129}$$

and

$$V = \frac{R}{1 + K F^2}. \tag{130}$$

In these expressions, $K(t)$ is the curvature parameter, $R(t)$ is the scale factor and $K_{,R} = K_{,t}/R_{,t} = dK/dR$; functions $f(r)$, $F(r)$ are defined accordingly three possibilities:

1. $f = r$, $F = r/2$;
2. $f = \sin r$, $F = \sin(r/2)$;
3. $f = \sinh r$, $F = \sinh(r/2)$.

Also it is assumed the energy-momentum tensor expressed above. What is very interesting here is the transformation that relates the radial coordinate r to the Stephani radial coordinate, say

$$r = \int \frac{d\tilde{r}}{1 + k_0 \tilde{r}^2/4}, \tag{131}$$

for $k_0 = 0, \pm 1$. Setting the ansatz (128) combined with the perfect fluid expression into Einstein's equations we obtain the time-time component of field equations

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{(K + k_0)}{R^2} = \frac{8\pi G}{3}\rho, \tag{132}$$

in which

$$
\begin{aligned}
&k_0 = 0, && f = r, && F = r/2; \\
&k_0 = 1, && f = \sin r, && F = \sin(r/2); \\
&k_0 = -1, && f = \sinh r, && F = \sinh(r/2).
\end{aligned}
$$

An extensive derivation of some observational quantities which describe the evolutionary kinematics of the Stephani universe, such as Hubble parameter and deceleration parameter, can be found in reference [14]. From such derivations, for an inhomogeneous Stephani model featured by a time dependent curvature index, with solely dust as the filling up fluid component (an oversimplification hardly acceptable), it was reported in the above reference an age of the universe notably larger than the estimated age provided by FLRW models with no exotic matter. Also, being this model formally described by dust-like matter, the curvature term is such that it simulates an exotic fluid driving the power-law inflation occurred at a later time.

Lastly, as in cosmology the redshift-magnitude relation is a key measure, I recommend to readers the formalism of power series around the observer's position for finding that relation proposed by Ellis and MacCallum [11].

4 Final remarks

Supergravity is, of course, a very attractive theory in the sense that, as pointed out by Wess , we may say that we understand a given system if we find a symmetry (or a supersymmetry) in the dynamics of this system [54]. For instance, in terms of canonical commutation relations, supersymmetry reads the energy momentum density tensor as a spin 2 object which is the graviton. But, even if we accept the recognized symmetries as de additional dimensions constituting the inner space of the system, the inexorable fact is that those supersymmetries remain year by year an experimental hope, perhaps during a never ending wait.

One thing I learned as a theoretical physicist is that one can never blindly accept a model as much as we like it. During last decades, theories have become more mathematical than physical, in part because we are dealing phenomenologically with a reality difficult to access empirically, and this requires us to be much more cautious in our reflections on the validity of our representations. I was particularly happy to see that from my first readings on quantum gravity, the same author who impressed Rovelli , Chris Isham , also caught my attention. Since then, I never stopped to review my own doubts and concerns about quantum gravity. I think that, at a given moment, I questioned my position on the supergravity theories, but not properly abandoning them, and this is what led me to the formulation of my quantum approach of the spacetime. Indeed, such an approach is still necessarily phenomenological, but at least it does not raise extra dimensions, nor requires the acceptance of hypothetical particles, being compatible with general relativity.

The study of the theories of gravity plays a fundamental role in the progress of human understanding on cosmology. In particular, the physics of gravitational singularities, such as black-holes and the former Big-Bang, certainly has the quantum-mechanical key to shed lights on the intersection between GR and QM within the framework of the modern cosmological theories. Indeed, there is still much speculation about the physics of black holes and Einstein's bridges (the original denomination for wormholes). For instance, Maldacena and others consider the possibility of entanglement between two black-holes, giving rise to a wormhole, that is, a "conduit" shared by both [24]. While it still takes a long time to reach a clearer picture of the fourth interaction, personally I do not think that a true unification of fundamental interactions is possible, but just a unification of general principles through a «master» principle. Such unification is in sharp progress (although at certain moments in a somewhat confusing way) because we already have that master principle: the principle of gauge. As we have seen, this principle is so powerful that we can appreciate it in classical theories as thermodynamics applied to engineering systems; so profound that we can see it sprout naturally in every phenomenology of the smooth transformations. From Weyl to Rovelli, through Fock, Lyra, Yang, Mills and O'Reifeartaigh, the gauge theories remain the most beautiful and effective theoretic tools that the reason has produced at all times.

Quanto mais fundamente penso, mais
Profundamente me descompreendo.
O saber é a inconsciência de ignorar...
Fernando Pessoa

CHAPTER II

Quantized spacetime *versus* supergravity

The first condition to establish a connection between the two approaches is that supergravity must appear only in a G-closure with a core embedded into an adS spacetime surrounded by a de Sitter layer, a great conjecture still waiting for further investigations. Therefore, a complete and consistent explanation is beyond the possibilities for now. Nevertheless, to my knowledge, adS conjectures about bubbles of compressed spacetime have not yet been addressed in a phenomenological proposal. So, nothing forbids to do a preliminary study aimed at the outer region dominated by dS metric.

The adS background and the paradigm for G-closures

Anti-de Sitter geometries are studied for a long time. Several works are developed last decades based upon adS scenarios, mainly focusing on black-holes. For instance, Kichakova *et al.* [17], studying the thermodynamics of asymptotically adS black-holes, got the so-called hairy black-holes in a spherically symmetric frame as solutions (with magnetic fields only) of the Einstein-Yang-Mills-SU(2) equations for a negative cosmological constant $\Lambda = -3/L^2$ within the ansatz

$$ds^2 = \frac{dr^2}{N(r)} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) - \sigma^2(r)N(r)dt^2, \tag{133}$$

where for the metric we have

$$N(r) = 1 - \frac{2m(r)}{r} + \frac{r^2}{L^2}, \tag{134}$$

with L as the adS radius. They particularized the approach for $\sigma(r) = 1$, satisfying some assumptions referring to a horizon located at $r = r_H > 0$ in order to write the Schwarzschild adS metric,

$$N(r) = \left(1 - \frac{r_H}{r}\right)\left(1 + \frac{r^2}{L^2} + \frac{rr_H}{L^2} + \frac{r_H^2}{L^2}\right), \tag{135}$$

and the embedded Abelian magnetic Reissner-Nordström-adS metric,

$$N(r) = \left(1 - \frac{r_H}{r}\right)\left(1 + \frac{r^2}{L^2} + \frac{rr_H}{L^2} + \frac{r_H^2}{L^2} - \frac{\alpha^2}{rr_H}\right), \tag{136}$$

in which

$$\alpha^2 = \frac{4\pi G}{\hat{g}^2}, \tag{137}$$

with $\hat{g}$ as the gauge coupling constant. Also, Alberghi *et al.*, in the early 2000s, gave us an interesting work on thermodynamics for radiating shells in adS spacetime where the spherical symmetry was divided into inner and outer regions separated by a thin massive shell [1]. The inner spacetime was represented by static coordinates according to the ansatz

$$ds_i^2 = -N_i(r)dt^2 + \frac{dr^2}{N_i(r)} + r^2 d\Omega^2, \tag{138}$$

and described by a Schwarzschild metric with

$$N_i(r) = 1 - \frac{2m}{r}. \tag{139}$$

The outer spacetime, due to the radiation from the shell, is described by a Vaidya-adS infinitesimal element

$$ds_o^2 = -\frac{1}{N_o(r,t)}\left[\left(\frac{\partial_t M(r,t)}{\partial_r M(r,t)}\right)^2 dt^2 - dr^2\right] + r^2 d\Omega^2, \tag{140}$$

with

$$N_o(r,t) = 1 - \frac{2M(r,t)}{r} + \frac{r^2}{L^2}, \tag{141}$$

where $M(r,t)$ is the Bondi mass depending on the time because of its relation to the amount of energy flowing out of the shell as radiation. Analyzing particle production in adS spacetime, Greenwood *et al.* conducted a study with negative cosmological constant with a solution for the scale factor given by $R(t) = H^{-1}cos(Ht)$ where $H = \sqrt{|\lambda|/3}$. In this case, the time-dependent adS metric took the same form as dS metric

$$ds^2 = -dt^2 + R^2(t)\left(dr^2 + r^2 d\Omega^2\right), \tag{142}$$

except for the scale factor $R(t)$. According to the authors, for the decaying segment of $cos(Ht)$, this time-dependent solution describes a collapsing universe under the influence of the negative constant vacuum energy density [13]. Many more examples could be given elsewhere [3], [43]. In particular, for further investigations, I consider an inverse situation of Alberghi's modeling, doing the core of the G-closure described by the Vaidya-adS ansatz given in equation (140), since radiation flows from dS external spacetime to the inner region due to the impacts of gravitons and the consequent mass intermediation mechanism between the two spacetimes. However Greenwood's picture seems the simplest representation for the adS central region of the G-closure, it has the inconvenient to describe a collapsing situation, which is not the case of my proposal where spacetime is under compression, not in imploding process.

Indeed, in this scenario mass is an effect of transition between adS and dS spacetimes. Since some G-closures, at least in principle, are now supposed to be formed by an adS core covered by a dS layershell, we may focus on the dS spacetime to analyze the compressed region between two massive bodies in a binary system, leaving aside for further study the characteristic phenomenology of the adS region.

The external de Sitter spacetime

From now on, I shall concentrate on presenting the latest theoretical results from my treatment of the spacetime quantum structure, giving a brief on the first ideas.

The second form of the geodesic equation in singularity functions was deduced in previous work [46] as

$$\frac{d^2\langle x-\varepsilon\rangle_\xi}{d\tau^2}+\Gamma^\xi_{\mu\nu}\frac{d\langle x-\varepsilon\rangle_\mu}{d\tau}\frac{d\langle x-\varepsilon\rangle_\nu}{d\tau}=0, \tag{143}$$

where the affine connection is

$$\Gamma^\xi_{\mu\nu}=\frac{\partial\langle x-\varepsilon\rangle_\xi}{\partial\chi^\eta}\frac{\partial^2\chi^\eta}{\partial\langle x-\varepsilon\rangle_\mu\partial\langle x-\varepsilon\rangle_\nu}. \tag{144}$$

One must understand that G-closures are bubbles of spacetime where the continuum itself is under gravitational compression, making spacetime in permanent contraction, not in expansion. This is like an elastic sheet being stretched in all directions except in a small area in which it was being shortened by the action of an opposing force that locally shrinks the sheet likewise in all directions. When space contracts, the scale reduces and it appears what I call "translational coupling", that is, a coupling between translation and contraction in such manner that the final result is no real displacement. In other words, the contraction of space compensates any displacement, so that the geodesic equation reduces to

$$\frac{d^2\langle x-\varepsilon\rangle_\xi}{d\tau^2}+\Gamma^\xi_{00}\frac{d\langle x-\varepsilon\rangle_0}{d\tau}\frac{d\langle x-\varepsilon\rangle_0}{d\tau}=0; \tag{145}$$

$$\frac{d^2\langle x-\varepsilon\rangle_\xi}{d\tau^2}=-\Gamma^\xi_{00}\frac{d\langle x-\varepsilon\rangle_0}{d\tau}\frac{d\langle x-\varepsilon\rangle_0}{d\tau}. \tag{146}$$

Since no effective displacement occurs, field becomes static in space, so that

$$\Gamma^\xi_{00}=\frac{1}{2}g^{\xi\lambda}\left(\partial_0 g_{\lambda 0}+\partial_0 g_{0\lambda}-\partial_\lambda g_{00}\right) \tag{147}$$

reduces to

$$\Gamma^{\xi}_{00} = \frac{1}{2} g^{\xi\lambda} \left(\partial_0 g_{\lambda 0} + \partial_0 g_{0\lambda} \right). \tag{148}$$

Now, supposing local isotropy, we can introduce a locally de Sitter classical background on a manifold which admits flat 3-sections. To find the invariant measure of the spacetime contraction rate, within any interval of the geodesic line, firstly we write the invariant element in commoving coordinates by the correlation function

$$\begin{aligned} \langle 0| \, g_{\mu\nu} d\langle x-\varepsilon\rangle_{\mu} d\langle x-\varepsilon\rangle_{\eta} \, |0\rangle &= -d\langle t-\varepsilon\rangle_0^2 \\ &+ R^2_{\langle t-\varepsilon\rangle_0} d\left\langle \mathbf{x}-\boldsymbol{\varepsilon}\right\rangle d\left\langle \mathbf{x}-\boldsymbol{\varepsilon}\right\rangle, \end{aligned} \tag{149}$$

in which we have the effective Huble constant as the logarithmic derivative of the scale factor

$$H_{eff} \equiv \frac{d}{d\left\langle t-\varepsilon\right\rangle_0} \ln\left(R_{\langle t-\varepsilon\rangle_0} \right). \tag{150}$$

Comparing expression (149) with the quantum-corrected invariant element in conformal coordinates, we get

$$\begin{aligned} &\langle 0| \, g_{\mu\nu} d\langle x-\varepsilon\rangle_{\mu} d\langle x-\varepsilon\rangle_{\eta} \, |0\rangle \\ &= \Omega^2 \left\{ -\left[1-C(u)\right] du^2 + \left[1+A(u)\right] d\left\langle \mathbf{x}-\boldsymbol{\varepsilon}\right\rangle d\left\langle \mathbf{x}-\boldsymbol{\varepsilon}\right\rangle \right\}, \end{aligned} \tag{151}$$

where u is a time function that corresponds to $1/H$ for time coordinate equal to 0 and to 0 for time coordinate equal to ∞. The quantities $A(u)$ and $C(u)$ are defined from the retarded Green's functions of the massless minimally coupled and conformally coupled scalars, so that

$$A(u) = -4G^{ret}_A \left[a\right](u) + G^{ret}_C \left[3a+c\right](u), \tag{152}$$

$$C(u) = G^{ret}_C \left[3a+c\right](u). \tag{153}$$

From the above unfoldings, we can deduce the scale factor from the quantum-corrected invariant element in conformal coordinates [55], and relate u to t

$$R_{\langle t-\varepsilon\rangle_0} = \Omega\sqrt{1+A(u)}, \tag{154}$$

$$d\left\langle t-\varepsilon\right\rangle_0 = -\Omega\sqrt{1-C(u)}du. \tag{155}$$

Then, for the perspective of finding a G-closure, we must write

$$\langle 0| \, g_{\mu\nu} d\langle x-\varepsilon\rangle_{\mu} d\langle x-\varepsilon\rangle_{\nu} \, |0\rangle = -d\langle t-\varepsilon\rangle_0^2, \tag{156}$$

for a timelike geodesic, and then

$$\langle 0| \, g_{\mu\nu} d\langle x-\varepsilon\rangle_{\mu} d\langle x-\varepsilon\rangle_{\nu} \, |0\rangle = \Omega^2 \left\{ -\left[1-C(u)\right] du^2 \right\}. \tag{157}$$

In short, we can only predict the expectation value of the rate in which the invariant element evolves in time mode, once a G-closure is manifested. As already emphasized, such considerations were madden to match quantum spacetime with quantum Riemannian metric as a way to quantize not the gravitational field, but the spacetime on its own, establishing compliance between quantum gravity and Einstein's general relativity within a unique geometric framework. From expressions (154) and (155), assuming $\Omega = 1/Hu$ in de Sitter geometry, I have obtained

$$\frac{du}{d\left\langle t-\varepsilon\right\rangle_0} = -\frac{1}{\Omega\sqrt{1-C(u)}} = -\frac{Hu}{\sqrt{1-C(u)}}, \tag{158}$$

and so

$$\frac{d}{d\left\langle t-\varepsilon\right\rangle_0}\left(\ln R_{\langle t-\varepsilon\rangle_0}\right) = -\frac{Hu}{\sqrt{1-C(u)}}\left[-\frac{1}{u}+\frac{1}{2}\frac{1}{1+A(u)}\frac{dA(u)}{du}\right] = \frac{H}{\sqrt{1-C(u)}}\left[1-\frac{1}{2}\frac{u}{1+A(u)}\frac{dA(u)}{du}\right]. \tag{159}$$

Determining general function $A(u)$ for very high densities from Green's formalism

As we have seen, Green functions appear crucially in the theory. In former works, Green's original conception was directed to electrostatic problems in bounded regions, where the Green function $G(r, r')$ is the potential at the point r produced by a unit point charge at r'. Now, an easy illustration of Green function is given by a force $F(t)$ acting for a very short time on a particle initially at rest; the force is such that the corresponding impulse on the particle is chosen to induce a unit change in momentum at a time t'. Further, at time t, the displacement $s(t)$ of the particle is said to be the Green function $G(t, t')$. The global particle motion is obtained by integrating, from the initial time t_0 up to the time t, all the effects of the impulses applied, that is,

$$s_g(t) = \int_{t_0}^{t} G(t, t')F(t')dt'. \tag{160}$$

Several applications of Green's function are available elsewhere, including a fundamental role in particle physics.

To understand our point, it is interesting to make a basic approach on Green functions so that later, by this way, we can well understand the phenomenology discussed. Green's function is now defined as

$$\frac{d^2G(u,\zeta)}{du^2} = -\delta(u-\zeta), \quad (0.5 \le \zeta \le 1), \tag{161}$$

which corresponds to the general non-homogeneous equation

$$\frac{d^2\wp}{du^2} = -f(u), \quad 0.5 \leq u \leq 1, \tag{162}$$

so that, for the same boundary conditions,

$$G(0.5, \zeta) = G(1, \zeta) = 0. \tag{163}$$

As known, delta function is null for all $u \neq \zeta$, from which we deduce

$$\frac{d^2 G(u, \zeta)}{du^2} = 0, \quad (u \neq \zeta). \tag{164}$$

This equation has solutions to the left and right of $u = \zeta$ denoted by

$$G(u, \zeta) = \begin{cases} au + b, & 0.5 \leq u \leq \zeta \\ a_1 u + b_1, & \zeta < u \leq 1 \end{cases} \tag{165}$$

where a, b, a_1 and b_1 are integration constants coming from the boundary conditions, that is,

$$\left.\begin{matrix} G(0.5, \zeta) = 0.5a + b = 0 \\ G(1, \zeta) = a_1 + b_1 = 0 \end{matrix}\right\} \therefore b = -0.5a; \;\; b_1 = a_1. \tag{166}$$

Therefore,

$$G(u, \zeta) = \begin{cases} a\,(u - 0.5)\,, & 0.5 \leq u \leq \zeta \\ a_1\,(u - 1)\,, & \zeta < u \leq 1 \end{cases} \tag{167}$$

To know the values of a and a_1, we must determine the behavior of the Green's function in the vicinity of $u = \zeta$ (the interval $|\zeta - \varepsilon, \zeta + \varepsilon|$ with center at $\delta(u - \zeta)$). Also in the limit $\varepsilon \longrightarrow 0$, the function must be continuous, and its first derivative discontinuous at $u = \zeta$. Thus,

$$G(\zeta - 0.5, \zeta) = G(\zeta + 0.5, \zeta), \tag{168}$$

$$a\,(\zeta - 0.5) = a_1\,(\zeta - 1)\,. \tag{169}$$

The jump of the derivative from one side of the point $u = \zeta$ to the other is given by

$$\int_{\zeta-\varepsilon}^{\zeta+\varepsilon} \frac{d}{du}\left(\frac{dG(u, \zeta)}{du}\right) du = -\int_{\zeta-\varepsilon}^{\zeta+\varepsilon} \delta\,(u - \zeta) du, \tag{170}$$

$$\left.\frac{dG(u, \zeta)}{du}\right| \begin{matrix} \zeta + \varepsilon \\ \zeta - \varepsilon \end{matrix} = -1, \tag{171}$$

$$G'(\zeta + \varepsilon, \zeta) - G'(\zeta - \varepsilon, \zeta) = -1. \tag{172}$$

When $\varepsilon \longrightarrow 0$,

$$G'(\zeta+0,\zeta) - G'(\zeta-0,\zeta) = -1. \tag{173}$$

Since

$$G'(u,\zeta) = \begin{cases} a, & 0.5 \leq u \leq \zeta \\ a_1, & \zeta < u \leq 1 \end{cases} \tag{174}$$

we have, according to equation (173),

$$a_1 - a = -1. \tag{175}$$

Now, replacing in equation (169),

$$\begin{aligned} &a\,(\zeta - 0.5) = a_1\,(\zeta - 1)\,, \\ &a\,(\zeta - 0.5) = (a-1)\,(\zeta - 1)\,, \\ &a = 2\,(1-\zeta)\,; \end{aligned} \tag{176}$$

$$\begin{aligned} &(a_1+1)\,(\zeta - 0.5) = a_1\,(\zeta - 1)\,, \\ &a_1 = 1 - 2\zeta. \end{aligned} \tag{177}$$

Therefore, the Green's function gets the form

$$G(u,\zeta) = \begin{cases} (1-\zeta)\,(2u-1)\,, & 0.5 \leq u \leq \zeta \\ (1-u)\,(2\zeta-1)\,, & \zeta < u \leq 1 \end{cases} \tag{178}$$

which is symmetric interchanging u by ζ. Once the Green's function has been determined, the solution of the equation (162) is obtained from the integration

$$\wp(x) = \int_{0.5}^{1} d\zeta G(u,\zeta) f(\zeta). \tag{179}$$

Now, when one speaks of the universe at the Big Bang vicinity, it is evident a fundamental property: its incredibly high density. One can only hope that such a density affects drastically the very evolution of the spacetime for a fleeting interval. It is worth to ask how the derivative of the function $A(u)$ is affected, that is, to what extent $dA(u)$ is inhomogeneous immediately after the Big Bang. An interesting way to get the answer is through Green functions. It is understood that $A(u)$ should assume a somewhat different form at the immediate surroundings of the Big Bang event. This reasoning obviously leads to second-order operations. Let us take equation (159), which near the Big Bang requires

$$1 - \frac{1}{2}\frac{u}{1+A(u)}\frac{dA(u)}{du} = 0, \tag{180}$$

or

$$\frac{dA(u)}{du} = -\frac{2\,(1+A(u))}{u}. \tag{181}$$

Independently of the global matter density time evolution, if we consider, by physical similarity, this expression valid in the imediate vicinity of a supermassive black-hole, we may ask how inhomogeneous is the detivative of $A(u)$ with respect to the time function u. Stated differently, we may determine function $A(u)$ at high densities for any interval of the time function u, taking into account that we can only evaluate the inhomogeneity of $dA(u)$ by applying a "less local" operator than the first order derivative. This "less local" operator is precisely the second order derivative [20]. Then,

$$\frac{d^2A(u)}{du^2} = -\left[\frac{2}{u}\frac{dA(u)}{du} - \frac{2}{u^2}\left(1 + A(u)\right)\right], \tag{182}$$

that is,

$$\frac{d^2A(u)}{du^2} = -f(u), \tag{183}$$

which has the form of equation (162). Given that Green's function on the left of the point ζ is different if taken on the right, we can write

$$\begin{aligned} A(u) = (1-u)\int_{0.5}^{u} d\zeta\,(2\zeta-1)\left[\frac{2}{\zeta}\frac{dA(\zeta)}{d\zeta} - \frac{2}{\zeta^2}\left(1 + A(\zeta)\right)\right] \\ + (2u-1)\int_{u}^{1} d\zeta\,(1-\zeta)\left[\frac{2}{\zeta}\frac{dA(\zeta)}{d\zeta} - \frac{2}{\zeta^2}\left(1 + A(\zeta)\right)\right]. \end{aligned} \tag{184}$$

To verify this solution we must apply the fundamental theorem of calculus and get

$$\begin{aligned} & A'(u) \\ &= (1-u)(2u-1)\left[\frac{2}{u}\frac{dA(u)}{du} - \frac{2}{u^2}\left(1 + A(u)\right)\right] - \int_{0.5}^{u} d\zeta\,(2\zeta-1) \\ &\times \left[\frac{2}{\zeta}\frac{dA(\zeta)}{d\zeta} - \frac{2}{\zeta^2}\left(1 + A(\zeta)\right)\right] - (2u-1)(1-u)\left[\frac{2}{u}\frac{dA(u)}{du} - \frac{2}{u^2}\left(1 + A(u)\right)\right] \\ &\quad + 2\int_{u}^{1} d\zeta\,(1-\zeta)\left[\frac{2}{\zeta}\frac{dA(\zeta)}{d\zeta} - \frac{2}{\zeta^2}\left(1 + A(\zeta)\right)\right]; \end{aligned} \tag{185}$$

$$\begin{aligned} A'(u) = -\int_{0.5}^{u} d\zeta\,(2\zeta-1)\left[\frac{2}{\zeta}\frac{dA(\zeta)}{d\zeta} - \frac{2}{\zeta^2}\left(1 + A(\zeta)\right)\right] \\ + 2\int_{u}^{1} d\zeta\,(1-\zeta)\left[\frac{2}{\zeta}\frac{dA(\zeta)}{d\zeta} - \frac{2}{\zeta^2}\left(1 + A(\zeta)\right)\right]. \end{aligned} \tag{186}$$

[20] For a while it was thought to represent a given nonlocal aspect by means of successive differentiations, which would surely imply in an infinite number of derivatives and, therefore, to an absolute indetermination. Indeed, each derivative produces a less local effect than the previous one, so that infinite derivatives are necessary to arrive at a nonlocal physical representation.

Therefore,

$$A''(u) = -(2u-1)\left[\frac{2}{u}\frac{dA(u)}{du} - \frac{2}{u^2}(1+A(u))\right]$$
$$-2(1-u)\left[\frac{2}{u}\frac{dA(u)}{du} - \frac{2}{u^2}(1+A(u))\right]$$
$$= -\left[\frac{2}{u}\frac{dA(u)}{du} - \frac{2}{u^2}(1+A(u))\right] = -f(x). \tag{187}$$

Some final comments

Supergravity is, of course, a very attractive theory in the sense that, as pointed out by Wess , we may say that we understand a given system if we find a symmetry (or a supersymmetry) in the dynamics of this system [54]. For instance, in terms of canonical commutation relations, supersymmetry reads the energy momentum density tensor as a spin 2 object which is the graviton. But, even if we accept the recognized symmetries as de additional dimensions constituting the inner space of the system, the inexorable fact is that those supersymmetries remains year by year an experimental hope, perhaps during a never ending wait.

One thing I learned as a theoretical physicist is that one can never blindly accept a model as much as we like it. During last decades, theories have become more mathematical than physical, in part because we are dealing phenomenologically with a reality difficult to access empirically, and this requires us to be much more cautious in our reflections on the validity of our representations. I was particularly happy to see that from my first readings on quantum gravity, the same author who impressed Rovelli [40], Chris Isham [6], also caught my attention. Since then, I never stopped to review my own doubts and concerns about quantum gravity. I think that, at a given moment, I questioned my position on the supersymmetric theories, but not properly abandoning them, and this is what led me to the formulation of my quantum approach of the spacetime. Indeed, such an approach is still necessarily phenomenological, but at least it does not raise extra dimensions, nor requires the acceptance of hypothetical particles, being compatible with general relativity.

My points were to make a comprehensive physical explanation of a model still in progress (not to lead a comparative analysis among other theories of spacetime quantization), making compatible with general relativity the Planckian dimensions of certain gravitational singularities where the short-distance quantum nature of spacetime becomes relevant. Lastly, I deduced the form of the specific relation between the expectation value of the expansion (or contraction) rate of the universe and the expectation value of the energy density as

$$\langle 0| \, g_{\mu\nu} d \left\langle x-\varepsilon \right\rangle_{\mu} d \left\langle x-\varepsilon \right\rangle_{\nu} |0\rangle =$$
$$-\left[\frac{3}{8\pi G u^2 \left\langle \rho_{\langle t-\varepsilon \rangle_0} \right\rangle}\left(1-\frac{1}{2}\frac{u}{1+A(u)}\frac{dA(u)}{du}\right)^2\right] du^2. \qquad (188)$$

Therefore, I compared quantum spacetime with quantum Riemannian metric, measuring the spacetime shrinkage rate at the compressed region, having in mind the energy density, and analyzing some consequences of the high density at the immediate vicinity of massive objects. Some important points about the relations of my proposal with the classical formalism of general relativity were discussed in great detail in reference [46].

Epilogue

The *Thesaurus* is a new compilation of the first Lecture Notes edition, February 2017. It is an extended version from many suggestions offered by the students at the XIV Summer School of CBPF – Brazilian Center of Physics Research, January 2017. As the scientific Editor of the Brazilience Journal of Engineering and Applied Physics, I decide to publish this version, not only to preserve a historical record of the work evolution sponsored by the Journal, but to honor the participants, P. V. Paraguassú, Victor H. Alencar, V. Valadão, Rui Aquino, M. P. Macedo and Patrick F. Alexandrino — now friends of mine —, and my old master, José Abdalla Helayël-Neto (the partnership of so many years continues!). For all of them, without whom this work would not succeed, I dedicate this special edition.

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Appendix I

Reflections on the nature of the movement

It is curious how relativity is admitted at the same time that bodies are still accepted as spatial objects moving within "another" space, apparently dissociated from them. The artifice of dividing the world into macro and micro physical images does not help to solve the paradox, since there must be a unique nature for everything, regardless of how humanity perceives it. I think the problem is solved by a very simple mental procedure. Imagine that, initially, the space was restricted to a point coincident with an elemental punctual being; moreover, it is appropriate that the term "expansion" refers to a representation consisting of a continuous fluxion of points (as physicists, we borrow from mathematics the objects necessary for the assembly of our constructs, without expecting that mathematicians understand us). Our punctual friend would not go anywhere confined to his sharp prison. Suppose that suddenly a point immediately adjacent to the first appears. Our hero could go back and forth from one point to the other, provided the continuity of the interval between the two points was guaranteed. In other words, it is only possible to go from one point to another if there is fluxion, that is, continuous expansion of the space between them, however close they may be (a static interval is an abstraction belonging to geometry, not to physics). At a certain moment, as new points are created, our hero (which is also constituted of space, and, therefore, is under the same laws of fluxion) does not realize that his own being likewise expands into new points, establishing a permanent scale ratio between it and its contiguous universe. Such point fluxion is virtually inexhaustible, so that to go from point A to point B is a feasible action simply because A and B are continually recreated along with the infinity of points separating them. Thus, it is the space that moves in and by itself.

Oddly enough, all this digression arose from a discussion of time machine effects that could occur under extreme gravitational conditions [44]. In this context, the crucial question is: in what way could the exclusion of space be represented in a geodetic path without nulling the spatial components of the metric tensor? This could happen in an exotic region in which the expansion was counterfeited by an intense gravitational pressure. So, if we assume that there is displacement only because space is continually expanding everywhere and in all directions, then a force opposing this expansion necessarily opposes any attempt of displacement. The phenomenological model for this theory was based on the zone of conflict between two coorbitant massive black holes, which inevitably led to a quantum spacetime theory founded on the thesis of fluxions.

Although the idea of fluxions is enlightening as a principle, it does not lend itself in its original state to reporting useful quantities to the physicist. For this reason, it is much more interesting to talk about a rate of change of an infinitesimal interval of fluxion. Such an interval defines our quantum of spacetime.

Appendix II

Understanding entanglement

It was pointed that quantum processing was born from "purely philosophically motivated questions" (Walther, 2006)[21] on non-locality and completeness of quantum mechanics fomented mainly by Einstein from his collaborative work with Podolsky and Rosen in 1935. In fact, as once observed, it was Einstein whom restored in modern science the Cartesian metaphysical sense of philosophy, turning physics into a real theory of knowledge (Charon, 1967)[22] . This important note remembers to us that philosophy will always be present in the process of creation. It is precisely its absence that determines little creativity that prevails today in all fields. Thus, to understand what is entanglement it will be necessary a reflective process of reconstruction of the conceptual foundations of physics, which will lead to a comprehensive review of the applicability of the notion of causality.

The main controversies of quantum mechanics ever resided in the difficulty of the human mind to separate the physical fact from its perception or representation. Indeed we always work with our perceptions; we took from them the full potential of human development and survival offered, creating representations for all we observe. There was a time when I was a follower of a kind of fruitless and paralyzing materialism that insisted to reify the world. Later, influenced by some physicists adepts of the operationalism, I came also to sympathize with the dresser and foolish idea that the only thing that matters is the calculation and not the ultimate nature of things. Thanks to my growing interest in quantum computing, I could deepen those controversial discussions and reach my own conclusions about them. Of course, long before the seventies there were eloquent speeches from the great thinkers of modern physics. Weizsäcker , for instance, in the Spanish version of 1974: *El átomo no es inmediatamente perceptible para nuestros sentidos, y cualquier experimento lleva sólo una determinada propiedad del átomo al ámbito de una perceptibilidad mediata* (Weizsäcker, 1974)[23]. But that was still little; not just to observe a predicate and describe it by means of classical concepts. It was necessary a phenomenal texture made by the experimental apparatus from which one could then

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extract useful measurements (information). In this it would lie a deepening of the famous complementarity of Bohr: the ultimate hidden object and its accessible and inseparable image.

Inspired by those philosophical texts from the first half of the twentieth century and early second half, I could refine my ideas and reach an understanding which I consider acceptable, although limited by the nature of human thought. Now I believe that the understanding of the quantum entanglement, one of the most intriguing phenomena of the quantum world, rises, for happiness of the philosophers, in a reflection on the edge of a pool. One summer night, I sat in a chair right in front of a lighted lamp whose flickering light was reflected in the pool. The image of the lamp stretched like a rubber with the ripples of the water and sometimes came to double or even to quintuple depending on the swings of the water. Both, the lamp and its images in water, are real, belonging to the world of mater and perceptions. But imagine that we could not see the lamp, only their images reflected in the water. We would think that two objects born of a unique (duplicate picture) would be irrevocably united, although separated; any change in one of them would "cause" an instantaneous change in the other. With respect to the quantum world is passing up something similar. We have no direct access to the ultimate reality (as the hidden lamp), only to the images produced by our experiments. What we see are the "pictures in the pool" and these are as real as the object that produced them. Clearly, these images carry information from the ultimate object, which makes them tractable to control. Instead of using the ultimate object we use them with all their informational potential. This potential is the base of the teleport process, since we teleport physical states, not matter in itself. In short, the quantum world is so light and sensible to our presence that it would be impossible to get direct benefits from their objects. All we can do is work with "pools". As Weizsäcker said: *Todo experimento es un acto material que es simultáneamente um acto de percepción* (Weizsäcker, 1974).

Appendix III

Epistemological considerations on paleogravity and its Lagrangian formalism

During my stay in Paris, I had the opportunity to discuss and refine my ideas about representation in physics, addressing aspects of the language we use and making critical considerations and suggestions on supergravity. In this appendix, I made a point of rewriting those suggestions in full in the way they were registered as a complement to the approach I have taken on paleogravity in the first part of this contribution.

Au cours de la fin du XXe siècle et au début de ce siècle, le débat sur la fin de la philosophie – la philosophie revisité comme une impasse, un «cul-de-sac» pour la compréhension humaine – retour à la mode, une sorte de «charme sec» intellectuelle révisionniste néo-moderniste. Heureusement, derrière cette discussion stérile reste la seule raison, inexorable, claire et absolument nécessaire de la philosophie: permettre la construction et l'amélioration critique de nos idées sur le monde des choses extérieures et de notre existence, apportant des réponses et de confort à notre esprit agité. Dans quelle mesure cette tâche est complexe ou satisfaisante il est une question plus affecte a la métaphysique, étant donné que la compréhension humaine est finie et toujours incomplète.

La physique contemporaine progresse lentement. Ceci est largement dû à des contraintes technologiques, mais il y a aussi des contraintes de nature conceptuelle et théorique. Selon mon point de vue, l'une des difficultés que la physique est confrontée se réfère à un manque de ressources linguistiques et formelles clairement physiques, ce qui a conduit la recherche en physique théorique de plus en plus à un simple exercice mathématique. Je ne pense pas que ce soit le droit chemin. Si nous ne gardons pas l'accent sur la représentation, l'accent sera mis sur les mathématiques, et non pas sur la représentation qui est, après tout, l'objet du theoricien. En fait, tout au long de ma vie universitaire, j'ai trouvé des collègues qui ont affirmé ne pas avoir vu beaucoup de physique à l'école d'études supérieures en physique pendant qu'ils travaillaient leurs thèses. Voici un sujet pour lequel une intervention philosophique profonde est nécessaire dans le sens de la recherche d'un équilibre entre lang age et représentation.

Une discussion philosophique important en science physique concerne a la recherche de symétries pour construire des modèles descriptifs dans la moderne théorie des champs, en particulier en référence à son application à l'étude de la gravité. Les théories dites supersymétriques sont construites *à partir d'une symétrie hypothétique entre bosons et fermions, gravitons et gravitinos dans le cas de la gravité. La théorie supersymétrique de la gravité est appelé «supergravité», et a occupé la plupart du temps que je consacre aujourd'hui à la physique.*

Nul ne peut nier que la symétrie est vraiment un aspect fondamental de la perception humaine et de la nature elle-même. Les ailes des papillons, les flocons de neige, les bras de las galaxies spirales, sont autant d'exemples de cette symétrie universelle qui semble imprégner notre propre façon de penser. Donc, il ne est pas étonnant la recherche de symétries en physique comme les tuiles constitutives des images que nous créons de la réalité en tant que fondements théoriques des lois de conservation. Ce est ainsi que, conformément au théorème de Noether, si un système est invariant dans une transformation de symétrie donnée, alors les lois de conservation sont attendues et nous pouvons expliquer formellement ces lois. L'interconnexion entre loi de conservation et symétrie est si forte que avec la connaissance de la symétrie nous sommes convaincus que nous connaissons le système. En outre, la forme mathématique des constructions de représentation est telle que ces symétries sont préservées sous toute transformation que la théorie prédit. Donc, ma revendication est la suivante: graviton et gravitino sont partenaires de la quatrième interaction; cependant, nous n'avons jamais eu aucune confirmation de leurs existences ; mais, même ainsi, nous pouvons parler de ces choses non observées comme des éléments abstraits d'une symétrie supposée; en ce sens, on obtient un langage pour parler de ces référents (particules, champs et ainsi de suite) dont la syntaxe est déterminée par la symétrie assignée.

Ma question épistémologique est de savoir comment parler de supersymétrie en gravité, même s'il n'y a aucune preuve de l'existence de particules comme gravitons et gravitinos. Une façon intéressante est parler de symétries abstraites; c'est-à-dire, si connaître les symétries est connaître le système, alors il est raisonnable de construire les symétries qui doivent régir la gravité, sans tenir compte de sa structure réelle, corpusculaire ou autre quelconque. Ainsi, ce qui importe est la symétrie proposée comme «background» des phénomènes, symétrie qui précède toute matérialisation phénoménologique.

Le formalisme Lagrangien est un bon outil pour construire des cadres. Selon les idées cidessus, le Lagrangien de l'interaction doit décrire uniquement les interactions entre les composantes de la symétrie, à savoir «graviton» et «gravitino», disons

$$\mathcal{L}_{\text{int}} = -4\left(\partial_\tau \langle G| \int |g\rangle \, d\tau + \partial_\tau |g\rangle \int \langle G| \, d\tau\right) - \gamma\langle G|^2 - 2\,\langle G|\,|g\rangle - \gamma^- |g\rangle^2 \tag{189}$$

avec $\langle G|$ représentant l'aspect graviton de la symétrie et $|g\rangle$ l'aspect gravitino. Les intégrales temporelles indéfinies traduisent l'hérédité manifeste dans les processus d'interaction locaux, parce que la gravité prend beaucoup de temps pour mettre en place d'une manière remarquable. Les matrices γ constituent une algèbre de Clifford $\mathbb{C}_{3,2}$ selon

$$\begin{array}{l} \left(\gamma^0\right)^2 = 1, \gamma^0 = \gamma^{0\dagger}(hermitienne), \\ \left(\gamma^a\right)^2 = -1, (a = 1, 2, 3), \gamma^a = \gamma^{a\dagger}(a - hermitienne), \\ \left(\gamma^4\right)^2 = 1, \gamma^4 = \gamma^{4\dagger}(hermitienne), \\ \gamma^a\gamma^b = -\gamma^b\gamma^a, a \neq b. \end{array}.$$

Si l'on applique l'équation d'Euler-Lagrange par rapport à $\partial_\tau |g\rangle$

$$\partial_\tau \frac{\partial L}{\partial \partial_\tau |g\rangle} - \frac{\partial L}{\partial |g\rangle} = 0, \tag{190}$$

$$-4|g\rangle + 2|g\rangle + 2\gamma\langle G| = 0, \tag{191}$$

$$|g\rangle = \gamma\langle G|. \tag{192}$$

Le même raisonnement appliqué à $\partial_t \langle G|$ fournit

$$\langle G| = \gamma^- |g\rangle\,. \tag{193}$$

D'une certaine manière, la structure des équations traitées uniquement du point de vue des symétries incorporées constitue une sorte de semasiografie, un symbolisme de description/représentation avec une grande épargne de caractères et d'éléments constitutifs. Pour le formalisme de la théorie des champs, les «kets» employés résument le contenu formel des champs. En d'autres termes, la forme des champs doit respecter la symétrie déterminée par la densité Lagrangienne semasiographique.

L'application du théorème de Noether, à savoir, l'interprétation de la densité Hamiltonienne comme une fonction de courant d'un «fluide» continu, conduit, si l'on suppose que la différence entre les courants liés aux champs $\partial_\tau \langle G|$ et $\partial_\tau |g\rangle$ est constante, à l'importante expression d'équilibre

$$[\langle G|, |g\rangle] = \partial_\tau \langle G| \int |g\rangle\, d\tau - \partial_\tau |g\rangle \int \langle G|\, d\tau = 0. \tag{194}$$

Pour cela, il est nécessaire d'introduire une constante adimensionnelle égale à 1/3 dans les «kets» des champs, de sorte que la symétrie est parfaitement vérifiée. Il est important de noter que les champs définis dans cette approche sont transformés conformément à l'action d'une algèbre de Clifford anti-de Sitter pour des raisons liées au fait que les gravitinos vivent dans l'espace anti-de Sitter.

En résumé, cette représentation offre les avantages suivants:

– *Les symétries sont traités indépendamment des référents de la théorie.*
– *Les composants de la symétrie ne nécessitent pas, en principe, d'un caractère probabiliste.*
– *Les champs associées aux «kets» sont assujettis à la symétrie introduite, de sorte que les équations peuvent uniquement afficher les résultats qui reflètent cette symétrie.*

Appendix IV

Singularity functions

I would like to reproduce here some observations I made in a recent paper. In a way, oddly enough, I was motivated to use singularity functions from studies and classes that I taught on the analysis of distinct spatial segments under the action of different deforming strains in a beam (and not from readings that led me to recognize a special case for integer λ of the well-known generalized function (distribution) from the classic book of Gelfand and Shilov "Generalized Functions, vol.1", Acad.Press, 1964). According to Beer and colleagues, singularity functions have formerly been applied for structural engineering analysis of beams beneath complex loads [24], being the present representation in "brackets" due to English mathematician William Macauley (1853-1936), although credits for the method are assigned to both German mathematician Alfred Clebsch (1833-1872) and German civil engineer Otto Föppl (1854-1924). In my opinion, however, preconceptions in scientific circles obscured the merits of the true author of singularity functions, the English mathematician Oliver Heaviside (1850-1925). Subsequently, these functions were applied in a variety of situations including productivity scheduling analysis by Lucko [25]. Now, I apply them to try to understand quantum gravity, particularly in present work associated with Lyra's geometry.

The main advantages of singularity functions to treat spacetime are in the facts that a)- they describe phenomena based on geometry, b)- they capture any changes in time evolution, c)- they can include infinitely many spacetime segments in different states, d)- they can be rescaled by any factor, e)- they are independent of units, and f)- they are continuous, differentiable and integrable like common functions. In this way, it is possible to treat geodetic arc segments in a wide perspective of fluctuations be-

[24] Beer, F., Johnston, E., Dewolf, J., and Mazurek, D., Mechanics of Materials, McGraw Hill, New York, 2012.
[25] Lucko, G., Journal of Construction Engineering and Management 135 246 (2009).

tween timelike and spacelike trajectory states with no appealing to zero components of the metric tensor.

So, within my scope, singularity functions serve to build a geodesic equation that makes possible to analyze any singular interval of the curve, defining the nature of the geodesic itself in each particular situation. A singularity function, given in Macauley kets as $\langle x - x_0 \rangle^n$, obeys the rule

$$\langle x - x_0 \rangle^n = \begin{cases} (x - x_0)^n, x > x_0 \\ 0, x \leq x_0 \end{cases} \tag{195}$$

In addition, making $\langle x - x_0 \rangle = \mathrm{X}$, we write

$$\int \langle x - x_0 \rangle^n d\mathrm{X} = \frac{1}{n+1} \langle x - x_0 \rangle^{n+1} + C; \tag{196}$$

$$\frac{d \langle x - x_0 \rangle^n}{d\mathrm{X}} = n \langle x - x_0 \rangle^{n-1}. \tag{197}$$

Appendix V

Foundations of nonlocality

Nonlocal effects are well known in physics since the late nineteenth century from works of Emile Picard (1891) and Alfred-Marie Liénard (1898), even though, with respect to retarded potentials, there are arguments that seem to prove that Bernhard Riemann had already cogitated in 1858 to generalize Poisson equation introducing the retardation. Also, Emil Wiechert (independently of Liénard) and Vito Volterra, in the early twenty century, considered to include far-off interferences in different phenomena. The classical example of nonlocal physics was consolidated in electromagnetism from Maxwell's theories, going hereafter to modern field theories. An absolute local theory seems to me as absurd as the old idea of absolute space, so that, once my model is based on cumulative processes of gravity and includes retarded Green's functions, it appears to be very instructive to make a succinct approach of this issue with eyes on the analysis about nonlocality in gravitation.

As we know, the potential $\phi(t, \mathbf{x})$ due to an electric charge $q(t)$ at the origin is solution of

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = -4\pi \varrho.$$

This solution for general coordinates $(t,\, x)$ has the form (the static case)

$$\phi(t, \mathbf{x}) = \frac{q_{\left(t - \frac{x}{c}\right)}}{x}.$$

By simple adding of contributions we can get for a general charge distribution

$$\phi\left(t,\mathbf{x}\right)=\int\frac{\varrho\left(t',x'\right)}{\left|x-x'\right|}d^{3}x',\tag{198}$$

according to the superposition principle. It is important to note that the solution of this integral in the form of a retarded Green function can be obtained from different manners, including the evaluation of Green function by its Fourier transform. In addition, that solution carries the retardation effect which contributes hereditarily to the field at time t with preterit charge density characteristics[26] at the time

$$t'=t-\frac{\left|x-x'\right|}{c}.$$

Similarly, we can obtain for the vector potential $\mathbf{A}\left(\mathbf{x},t\right)$

$$\mathbf{A}\left(t,\mathbf{x}\right)=\frac{1}{c}\int\frac{\mathbf{j}\left(t',x'\right)}{\left|x-x'\right|}d^{3}x'.\tag{199}$$

For the potentials associated with a single electric charge at an arbitrary velocity, the so-called Liénard-Wiechert potentials – to which the charge density is a Dirac-type measure associated with the moving charge – it passes something entirely different, since the potential $\phi\left(\mathbf{x},t\right)$ is nomore given from q divided by the retarded distance, but insted

$$\phi\left(t,\mathbf{x}\right)=\frac{q}{4\pi\varepsilon_{0}}\frac{1}{\left[r\left(1-\frac{\mathbf{v.x}}{cx}\right)\right]_{ret}},$$

and similarly

$$\mathbf{A}\left(t,\mathbf{x}\right)=\frac{\mu_{0}q}{4\pi}\left[\frac{\mathbf{v}}{r\left(1-\frac{\mathbf{v.x}}{cx}\right)}\right]_{ret}.$$

The quantities enclosed in square brackets must be calculated at the source. This is so because the retardation time t' – the time at which the signal that the observer is receiving at time t left the moving point-charge – becomes mandatory the evaluation of the charge densit at different times for different segments of the charge configuration, so that we shall get a warped image of the total charge (since the potentials are magnitudes that propagate, Doppler type effects are expected). Denominators (in brackets) are deduced from the expression of t,

$$t=t'+\frac{x'}{c};$$

[26] This is an instance of retarded time, not special relativity, since this latter involves time dilatation.

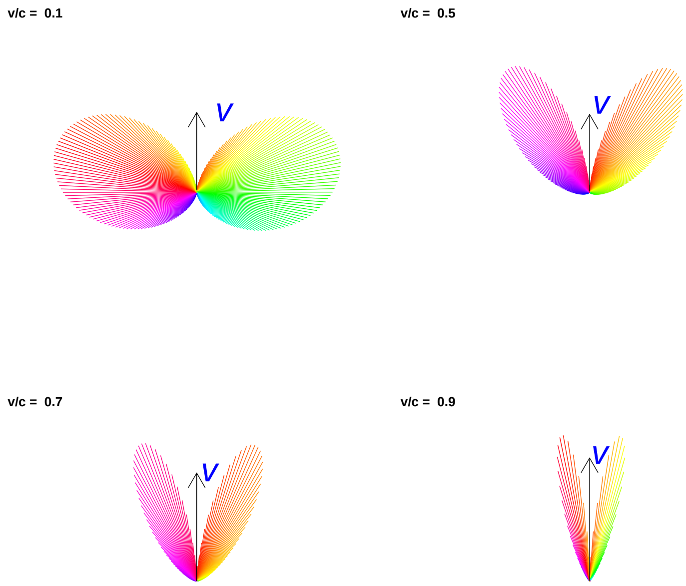

Fig. 3: Polar plots of the radiation intensity (acceleration $\parallel$ to v).

$$t = t' + \frac{x - x_{t'}}{c};$$

$$t' = t - \frac{x - x_{t'}}{c};$$

$$\frac{dt'}{dt} = 1 + \frac{1}{c}\frac{dx_{t'}}{dt'}\frac{dt'}{dt};$$

$$\frac{dt'}{dt}\left(1 - \frac{1}{c}\frac{dx_{t'}}{dt'}\right) = 1;$$

$$\frac{dt'}{dt} = \frac{1}{\left(1 - \frac{1}{c}\frac{dx_{t'}}{dt'}\right)},$$

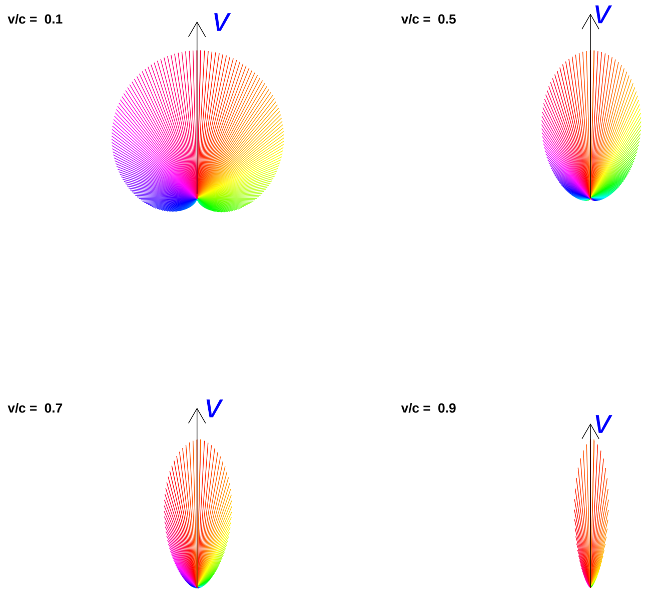

Fig. 4: Polar plots of the radiation intensity (acceleration $\perp$ to v).

where

$$\frac{dx_{(t')}}{dt'} = \frac{\mathbf{vx}}{cx}.$$

That warped image can be better understood by means of the angular distribution factor of the electric field during a *bremsstrahlung* (retard radiation), where the radiation diffuses not exactly in the direction of the velocity vector (Figures 1 and 2)[27]. From the point of view of cosmology, since the potentials of Liénard-Wiechert satisfy Maxwell's equations in Minkowski

[27] Note that in these pictures the colors tend to "cool" in the opposite direction of the velocity vector.

space, it might be interesting to study the irreversibility of radiative processes in face of the asymmetry of the expansion of the universe. This study is being considered in another proposal.

The potentials of Liénard-Wiechert are deduced from the retarded potentials measured in the observer,

$$\phi\left(t, \mathbf{x}\right) = \frac{1}{4\pi\varepsilon_0} \int dV' \frac{\varrho\left(t', x'\right)}{\left|x - x'\right|}, \tag{200}$$

$$\mathbf{A}\left(t, \mathbf{x}\right) = \frac{\mu_0}{4\pi} \int dV' \frac{\mathbf{j}\left(t', x'\right)}{\left|x - x'\right|}. \tag{201}$$

What is relevant here is the following: the retardation embedded in infinite sums means that the potentials at a given point P in a certain time t carry a nonlocality character, i.e., reflect the fact that the local electromagnetic effects due to the movement of an electric point-charge depends on the past experienced by the charge, suffering Doppler distortions because of the entry into scene of the charge velocity vector $\mathbf{v}$. In fact, the study of light ray deflection by the gravitational field of a moving mass shows the causal nature of gravity starting from the retardation of its effect which can be obtained theoretically applying the retarded Liénard-Wiechert model (Kopeikin & Fomalont, 2007). As in equations (198) and (199) for the static case, we can write

$$A\left(t, x\right) = G \int\limits_V \frac{\rho\left(t, x'\right)}{\left|x - x'\right|} d^3x' \tag{202}$$

and

$$\mathbf{A}\left(t, x\right) = \frac{G}{v_g} \int\limits_V \frac{\mathbf{j}\left(t, x'\right)}{\left|x - x'\right|} d^3x', \tag{203}$$

where ρ is the mass density and v_g is the speed of the gravitational field propagation.

However not corrected for quantum-mechanical effects, the Liénard-Wiechert potentials describe the electromagnetic nonlocal effects related to a moving point-charge in the Lorenz gauge (where for the vector potential $\mathbf{A}$, $\partial_\mu A^\mu = 0$, and Maxwell's equations reduce to the wave equation $\eta^{\mu\nu}\partial_\mu\partial_\nu A_\alpha = 0$).

Appendix VI

Final words for the sake of an innovative physics

I could not finish this study without making clear my understanding of how we should proceed in physics. I remember here that all my efforts as a theoretical physicist were never made without the permanent contribution from many logical-philosophical works of greatest importance such as those of Wittgenstein and Husserl, two of the thought giants of the twentieth century first half. After so many years of "goings and comings", now I think that a phenomenon "occurs" ("to occur" means to be in fact or to be logically imagined, like the phenomenological models), from the point of view of the human intelligence, not only in physical spacetime, but in the fusion of the physical spacetime with the mental spacetime. From such fusion it comes a representation, without which we cannot understand anything. Finally, a representation requires some logical substratum, which, in my opinion, is the symmetry (in fact, the brain always seeks patterns, or symmetries, to construct its models of understanding, and it is certainly aided and motivated by the very Nature, full of symmetries). In the modeling process, the variables of the subject are the forms used to describe the phenomenon in focus. Does all this mean that there is no reality outside consciousness? Not at all! What there is not is the understanding of the phenomenon without its abstract intellectual representation to be interpreted. In this way, we conceptualize several representatives, for which and among which we establish relationships in the form of a system of statements, before which we verify the reasonableness of the propositions we make about it (the system). It may be that this fusion becomes much more visceral if brain capacities are vastly amplified, but this is still mere speculation. I understand, therefore, that science is done by the projections of reality in our brains. We can never lose this perspective; otherwise we will lose the objectivity of physics in favor of obscure notions.

Index