Specimen Quaestionum Philosophicarum ex Scientia Physica Collectarum: Revisiting Leibniz in Physics with the Sharp-Product

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Abstract: This article shows the importance of semiological considerations in the writing of physics equations, in the sense of showing that certain multiplication operations must be indicated at first hand as main descriptors of the process represented by the equation, characterizing its nature. Only in this way will it be possible to interpret equations in their symbolic totality an correctness. Bringing some of Leibniz's ideas to the argumentative construction of the logical foundation of the central thesis, multiplication is discussed as an essential operation for the description of evolutionary processes, such as

$$\textit{one thing } \underbrace{[in-the-mode-of]}_{\times} \textit{ another } [equals] \textit{ new thing},$$

representing the necessary interactions for these processes. The notion of "sharp-product" is introduced to denote the multiplications that perform those interactions. It is hoped that, as a result of the discussion raised, the analysis of the mathematical formalizations of contemporary physics will be returned to their own ambit instead of getting lost in an abstractionism that does little to contribute to the formation of world representations.

Key words: semiology, multiplication, sharp-product, interaction, complex number.

Resumo: Este artigo mostra a importância das considerações semiológicas na escrita das equações da física, no sentido de evidenciar que certas operações de multiplicação devem ser indicadas como principais descritores do processo representado pela equação. Só assim será possível interpretá-las corretamente em suas totalidades simbólicas. Trazendo algumas ideias de Leibniz para a construção argumentativa da fundamentação lógica da tese central, discute-se a multiplicação como operação essencial para a descrição dos processos evolutivos, algo como

$$\textit{one thing } \underbrace{[in-the-mode-of]}_{\times} \textit{ another } [equals] \textit{ new thing},$$

representando as interações necessárias a estes processos. A noção de "sharp-product" é introduzida para denotar as multiplicações que desempenham aquelas interações. Espera-se que, como resultado da discussão levantada, a análise das formalizações matemáticas da física contemporânea seja gradualmente devolvida ao seu próprio âmbito ao invés de perder-se em um abstracionismo pouco contributivo para a formação das representações do mundo.

Palavras-chave: semiologia, multiplicação, *sharp-product*, interação, número complexo.

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Prologue

The mind, once stretched by a new idea,
never returns to its original dimensions.
R. W. Emerson

There is a fundamental difference between a researcher and a scientist. Not every researcher is scientist, although every scientist is a researcher. The difference is that the scientist does the research to establish the background of a new theory, a new system of hypotheses that answers an open question. The pure and simple researcher is usually content with citing and restating known results, sometimes applying alternative corroboration techniques. This is certainly an important work insofar as its results can reinforce or weaken a given theory or hypothesis, helping science to advance. But, a researcher would probably not be interested in something like the reconsideration of number theory or fundamental arithmetic, perhaps judging that there is nothing new to be learned in these matters. A scientist, on the contrary, would think of anything else to be said about the use of numbers and their basic operations as symbolic instruments for representing natural or anthropic processes. He is willing to deal philosophically with relatively simple things, however, with great syntagmatic[1] potential for the construction of new models and for a better interpretation of the ones that already exist.

Some of my first thoughts surfaced much later on matured theories in cosmology and thermodynamics. Long ago, when a young student, I had an inspiration and established a physical interpretation of the product operation between fields coupled by a constant. Shortly afterwards, I became aware of a Leibniz model as simple as mine, whilst in a wider purpose. Obviously, the title of this article pays tribute to Leibniz, placing him at the beginning of everything. Leibniz, as is known, was a true genius, an exuberant and incredibly creative mind [1, 18] to whom is credited the introduction of the dot as a multiplication sign. Although he invented calculus without knowing that Newton — another admirable genius — had already done it, we owe him the notation used today. He advanced to the creation of calculating machines and a binary numerical system. His concern with symbology promoted unprecedented formal progress. Now, going a step further, I return to the question of symbology, however, with the extra meaning that it can accumulate to build and represent a fundamental physical process in the writing of an equation.

In essence, the model Leibniz illustrated had the same semantic intention of the proposition I discuss: animal $\times$ rational $=$ man; the "mode" operation as I named it at the time. Generalizing, lastly, an attribute with a given level of complexity times another at any level of complexity is equivalent to an attribute of a higher level of complexity than the original two. By higher level I mean a certain circumstance such that an object or event would not arise or occur without the interactions between the primordial objects.

From a purely arithmetic point of view, multiplying is nothing more than adding successively. The theory-ladenness of my proposition is that, in physics, if two functions, φ_1 and φ_2, are multiplied with each other, this means that φ_1 evolves "in the mode of" φ_2, and vice-versa with the same result if the context is Abelian. This is what I called "sharp-multiplication" or "sharp-product", i.e., $\varphi_1 \sharp \varphi_2 = \varphi_{1,2}$. A sharp-product — P-$\sharp$ (P-sharp) — originates a new physical object, not the result of a simple successive sum. Thus, all multiplications between variables or functions in physics are sharp-products, as well as geometric operations with real magnitudes (ex: width $\sharp$ length $=$ area). It seems obvious, but notice that, when writing an equation representing a physical process, it is important to know which properties, which magnitudes must combine, inflecting each other and determining the paramount features of the process. Of course, equation terms are naturally identified by their roles: "this term does this...that term does that...", and so on. But what does each term actually summarize? Why introduce representations in complex numbers and their conjugates? Furthermore, how intuitive-deductive guidelines take part in the intellectual constructive process of an equation? And how does all this conspire for the success of a model? I think that dealing with equations of physical significance requires much more than simple mathematical manipulations. The meaning of what we are doing and the result we want

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[1] A linguistic critique of the symbolic forms we use and the definition of *syntagma* as a metaphysical element whose meaning depends on their relationships with other elements is available in reference [14].

to obtain depends on correct answers to those questions. When we say that two fields are coupled, we mean that one is inflected by the other, the intervening coupling constant being mainly a convenient dimensional adjustment factor. This P-♯ is in fact the representation of an interaction between two objects that gives another object. But what is the true content of such an operation and what we want with that?

The essentials

For simplists, science is all about finding ways to fit theories to empirical facts. This is certainly the route of least effort, for which we pay the price of superficiality and obscurantism. As Zinkernagel pointed out,

"[...]*science is often concerned with what the studied theories implies for our deeper understanding of the world. This involves the philosophical activity of interpreting the theories in question, and philosophy thus continues to be an integral part of scientific, including cosmological, thought.*"[17].

An example of dim ideas in physics proceeds from the fact that underdetermination in cosmology is often about solutions of general relativity, and, with regard to the expansion of the universe, the complication arises precisely because it is treated in terms of finding a natural way to superpose solutions in general relativity, something that does not exist, rather than assuming expansion — independently of mass — as a primigenous feature of the space-time continuum and not an aspect depending on solutions of equations[2] .

From this, as Ellis well emphasized with the scientific seriousness that is peculiar to him, cosmology as the territory of a philosophy about our existence and our place in the cosmos leads to questions far beyond the simplistic view, involving the emergence of complexity, meaning, purpose and ultimate causation [5]. In view of my complete agreement with this position, I can say that the physics I advocate — referring in particular to cosmology — starts from two basic philosophical principles (Leibniz influence!):

- The sufficient reason, i. e., "there must be sufficient reason for something to exist, for an event to occur, for a truth to be obtained" (It is important to remember that among the great priest cosmologists of the 20th century, God certainly appears as the sufficient reason for the greatness and harmony of the Universe). From the point of view of science, I evoke this principle in the foundations of contemporary cosmology in the following way: "The sufficient reason to assume the continuity of space-time is that it expands without sufficient reason to consider it discontinuous". This principle, arguably, poses serious objections to the idea of quantizing gravity.

- *natura non saltum facit*, a principle that is established as a corollary of the previous principle: the continuity of space-time in all things.

Besides, as defended by Leibniz, symbols are important for human understanding. Semiology is fundamental for the representations of physical processes to be elegant, clear and informative. So, good notations combining characters for simpler thoughts are the "encaustic tiles" to form complex thoughts.

Therefore, in physics, a product between two quantities is indeed a P-♯. Using a singular metaphor, it

[2] Recently, a local periodical published a small article (names don't matter here) in which the author, possibly delighted with so much data coming from all over, seemed to adopt a generalized position against the way theoretical work has been carried out in cosmology. In his partially pertinent criticism, he asks why we don't use the available data in our models, instead of spending time with mathematical exercises in directions that have already been rejected by observational science. On the whole, I agree, especially considering the time wasted looking for new solutions to Einstein's equation that suggest alternative universes. But the discourse conducted makes us believe that we are "swimming armfuls" in evidences, when in fact there is much controversy about critical points for the survival of the currently accepted model of the universe. Yes, the standard model remains the basis for essentially all research in cosmology, being the best we have so far, which does not exempt it from carrying serious problems that may oblige us to review our ideas from the beginning. If we take the issue of how fast the universe is expanding, the results have been found to be appallingly inconsistent. The two methods to measure Hubble constant — one based on early universe measurements, the other on measurements from nearby stars — continue to yield different values after decades of persistent work, and the problem was alarmingly worsened with new more accurate data from the Webb, which makes us suspect of a flaw in the current model. Faced with the fact that something may be really wrong, the standard model is now in check, forcing us to reevaluate our own ideas about the ultimate structure of the universe, something I have been trying to do.

comes to "tuning an equation a semitone up"!, indicating the multiplications critical to the overall meaning of the described process; that is, indicating in the equation who commands the process and who stabilizes it under specific conditions. Thus, when analyzing the content of the expressions, depending on the immanent complexity, we can rewrite them in a more instructive way, "tuning" them in P-$\sharp$. Firstly, my aim is to discuss the particular situation of conjugate products between classical fields represented in the complex numerical system. In this particular case, defining restrictively as sharp-applications the products between complex fields and their conjugated temporal derivatives seems sufficient from the semantic-ontological perspective in view of the nature of the fields in question, the fields of thermal energy I called "caloric fields". It would take a lot of time and a lot of useless discussion to decide to change the symbol for multiplication in physics, but it would be worthwhile to write the equations using the symbol "$\sharp$" (sharp) to indicate the applications between the fields leading to essential results that occupy a prominent place for the understanding of the theory. The sharp-application is the trigger of a creation; we therefore want to know what happens when one thing evolves in the manner of another thing in evolution:

one thing $\underbrace{[in-the-mode-of]}_{\times}$ *another* $[equals]$ *new thing.*

Of course, if powers were assigned to the product factors, we are symbolizing the weight each factor contributes to the final result.

: NEW INSIGHTS ON MORPHOLOGY OF THE PHYSICAL REPRESENTATION

P-$\sharp$ in caloric field theory

It is not my purpose here to discuss thermodynamics, something I have already done extensively in previous works. From the vast literature on thermodynamics, I highlight the references [2, 3, 4, 6, 8, 10, 15, 16]. I bring my own example along with other from cosmology, also discussed earlier, to consolidate the defended semiology.

From a macroscopic-cosmological point of view, thermodynamics stands out in the conception of a relativistic space-time woof in accelerated expansion as an ingredient theory, provided that it is considered in a new consistent perspective of the Second Law and, consequently, of the very concept of entropy. As Bejan pointed out, and here very much with regard to cosmology,

"*The universal principle of evolution belongs in thermodynamics because thermodynamics is a universal science and evolution is a universal phenomenon.*" [3, 4] .

My studies on contemporary thermodynamics and its broad application, including cosmology, leaded to the so-called "caloric field theory", comprising, within the scope of the work, approaches on the shape of the caloric field (the function that characterizes thermal energy itself) and the physical law of its propagation with the entropic trail left by the diffusion process. From a deep revision of the Second Law, in caloric field theory I defined entropy acceleration[3] as

$$\frac{\partial^2 S}{\partial \tau^2} = -2\gamma^2 \frac{\partial^2}{\partial \tau^2} \int |\xi|^2 \ln |\xi| \, dq$$

[3]Despite the insistence on adopting a Boltzmannian interpretation of the field entropy in generalized coordinates q given by

$$S = \int -2\gamma^2 |\xi|^2 \ln |\xi| dq, \tag{1}$$

associating it with state probabilities (I myself have already been subject to this interpretation), there is no sufficient reason for such an association, since entropy is a perfectly defined quantity in thermodynamics, without any stochastic connotation, as I have repeatedly stated in recent works. Since entropy irrevocably advances with time in our universe, much more consistent with the temporal arrow is to associate it with an evolutionary variable, a cosmic temporal marker in the continuous interval [0,1]. It is noteworthy that the differential equation of caloric fields,

$$\partial_q \partial^q \xi + \left(1-\gamma^2\right) \xi - \gamma^2 \xi \ln |\xi|^2 = 0, \tag{2}$$

with γ as an environmental constant, includes a field entropy term because the physical intuition is that the evolution of the field should expose the trace of its own entropy, since this magnitude accompanies the action of the field throughout its existence. Note that the meaning of an equation in physics transcends pure mathematical form; the expression (2) tells us that the evolution of the caloric field is concomitant with the entropy trail left by the field in action (1), and the environment imposes inflections to this evolution.

$$= -2\gamma^2 \frac{\partial^2}{\partial \tau^2} \int \xi\xi^\dagger \ln \sqrt{\xi\xi^\dagger} dq. \tag{3}$$

Some math manipulations involving partial derivatives led to

$$-2\gamma^2 \frac{\partial}{\partial \tau} \Big[\left(\dot{\xi}\xi^\dagger + \xi\dot{\xi}^\dagger\right) \ln \left(\xi\xi^\dagger\right)^{1/2} + $$

$$+ \xi\xi^\dagger \frac{1}{2(\xi\xi^\dagger)^{1/2}} \left(\xi\xi^\dagger\right)^{-1/2} \left(\dot{\xi}\xi^\dagger + \xi\dot{\xi}^\dagger\right) \Big]; \tag{4}$$

$$-2\gamma^2 \Big[\left(\ddot{\xi}\xi^\dagger + 2\dot{\xi}\dot{\xi}^\dagger + \xi\ddot{\xi}^\dagger\right) \ln \left(\xi\xi^\dagger\right)^{1/2} +$$

$$+ \left(\dot{\xi}\xi^\dagger + \xi\dot{\xi}^\dagger\right) \frac{1}{2(\xi\xi^\dagger)^{1/2}} \frac{\left(\dot{\xi}\xi^\dagger + \xi\dot{\xi}^\dagger\right)}{\left(\xi\xi^\dagger\right)^{1/2}} +$$

$$+ \frac{1}{2} \left(\ddot{\xi}\xi^\dagger + 2\dot{\xi}\dot{\xi}^\dagger + \xi\ddot{\xi}^\dagger\right) \Big]. \tag{5}$$

The reader can find the complete explanation in reference [10].

Now, of the ways a field in complex notation interacts with itself, perhaps the most significant is the interaction of the field with the time derivative of its complex conjugate. To take but one single example drawn from the formalism developed above, imagine a caloric field composed of two functions so susceptible to change over time that they are able to interact with their own rates of change (this type of interaction was called "*calorergy*", a kind of synergy between the propagation of heat and its own variation rate; indeed, I see this as a characteristic feature of complex systems). Furthermore, due to mutual interactions, each component function interacts equally with its partner's rate of change. Also, more than the thermal energy itself, it's the thermal energy differences that really make it all happen. Therefore, besides total *calorergy*, we are interested in expressing the difference between these later interactions, however separately, so that we can analyze in what proportion it reflects the evolution of the system as a whole. In a nutshell, all this is given by the P-♯,

$$\mathcal{A} : \xi \to \dot{\xi}^\dagger = \xi\sharp\dot{\xi}^\dagger = \xi\dot{\xi}^\dagger. \tag{6}$$

For the sake of brevity, let us take the field and its conjugate as

$$\xi = \varphi_1(t) + i\varphi_2(t),$$

$$\xi^\dagger = \varphi_1(t) - i\varphi_2(t),$$

with the time derivative of the conjugate given by

$$\dot{\xi}^\dagger = \dot{\varphi}_1(t) - i\dot{\varphi}_2(t).$$

The P-♯ is a simple product but exactly expressing the semantic prescriptions explained above in the form

$$\begin{aligned} &\xi\sharp\dot{\xi}^\dagger = \left[\varphi_1(t) + i\varphi_2(t)\right]\left[\dot{\varphi}_1(t) - i\dot{\varphi}_2(t)\right] = \\ &= \varphi_1(t)\dot{\varphi}_1(t) - i\varphi_1(t)\dot{\varphi}_2(t) + i\varphi_2(t)\dot{\varphi}_1(t) + \\ &+ \varphi_2(t)\dot{\varphi}_2(t) = \varphi_1(t)\dot{\varphi}_1(t) + \varphi_2(t)\dot{\varphi}_2(t) + \\ &+ i\left[\varphi_2(t)\dot{\varphi}_1(t) - \varphi_1(t)\dot{\varphi}_2(t)\right]. \end{aligned}$$

In the last equality, the right-hand side shows the imaginary unit as nothing more than a separator, isolating the sum of the interactions between the field components and their respective rates of change, and the difference of the interactions between the field components and their respective partner's rate of change, as specified in natural language. Without the search for this understanding, all the effort of calculation is a simple mathematical exercise with no clear physical meaning.

If we consider that the first-order interactions are the most relevant for characterizing the sensibility of the functions that form the field, it is enough to rewrite equation (5) as

$$-2\gamma^2 \Big[\left(\ddot{\xi}\xi^\dagger + 2\dot{\xi}\dot{\xi}^\dagger + \xi\ddot{\xi}^\dagger\right) \ln \left(\xi\xi^\dagger\right)^{1/2} +$$

$$+ \left(\dot{\xi}\sharp\xi^\dagger + \xi\sharp\dot{\xi}^\dagger\right) \frac{1}{2(\xi\xi^\dagger)^{1/2}} \frac{\left(\dot{\xi}\sharp\xi^\dagger + \xi\sharp\dot{\xi}^\dagger\right)}{\left(\xi\xi^\dagger\right)^{1/2}} +$$

$$+ \frac{1}{2} \left(\ddot{\xi}\xi^\dagger + 2\dot{\xi}\dot{\xi}^\dagger + \xi\ddot{\xi}^\dagger\right) \Big]. \tag{7}$$

The general theory of caloric fields associates time, energy and entropy without any stochastic demand, from a perspective of permanent thermal evolution of the systems. As discussed earlier in cosmology [11, 14], an interesting thing when thinking about this association is that an expanding sub-Planckian temporal interval can be understood as a reservoir of thermal energy in entropy flux, just as an expanding spatial interval can be assumed as a container of a certain amount of matter in dispersion. This way of describing the cosmic woof is a natural outcome of the presumed unity between space, time and gravitation, a concept that leaves no doubt about the physicality of time. As will be seen shortly, this has a decisive impact on understanding the structure of a space-time geodesic.

P-♯ in gravitational driving

Another example comes from the expression of the Euler-Lagrange modified sine-Gordon type equation of the geodesic line referring to the metric $ds^2 = -e^{2\phi(\mathrm{X}^1,\mathrm{X}^2,\mathrm{X}^3)}dt^2 + \zeta_{ij}(\mathrm{X}^1,\mathrm{X}^2,\mathrm{X}^3)d\mathrm{X}^i d\mathrm{X}^j$, in which an arbitrary interval subject to expansion or contraction is warped by a soliton of acceleration, a solitary gravitational pulse, [11], say

$$\frac{d}{ds}\left(\zeta_{ij}\widetilde{\mathrm{X}}^j\right) + \frac{\partial\phi}{\partial\mathrm{X}^i}e^{2\phi}\tilde{t}^2 - \frac{\partial\zeta_{jk}}{\partial\mathrm{X}^i}\frac{\widetilde{\mathrm{X}}^j\widetilde{\mathrm{X}}^k}{2} + +m^2\tilde{t}E\sin\vartheta\frac{\partial\vartheta}{\partial\mathrm{X}^i} = 0, \tag{8}$$

with the corresponding Lagrangian density

$$\mathcal{L} = \frac{1}{2}\left(-e^{2\phi}\tilde{t}^2 + \zeta_{ij}\widetilde{\mathrm{X}}^i\widetilde{\mathrm{X}}^j\right) - m^2\tilde{t}E\left(1-\cos\vartheta\right) \tag{9}$$

for

$$\frac{d}{ds}\left(\frac{\partial\mathcal{L}}{\partial\widetilde{\mathrm{X}}^i}\right) - \frac{\partial\mathcal{L}}{\partial\mathrm{X}^i} = 0. \tag{10}$$

The arbitrary constant E matches the freedom of the null geodesics affine parameter and is interpreted as the expansion energy contained in the worldline intervals [11]. In this case, with P-♯ terms equation 8 is rewritten as

$$\frac{d}{ds}\left(\zeta_{ij}\sharp\widetilde{\mathrm{X}}^j\right) + \frac{\partial\phi}{\partial\mathrm{X}^i}e^{2\phi}\tilde{t}^2 - \frac{\partial\zeta_{jk}}{\partial\mathrm{X}^i}\sharp\frac{\widetilde{\mathrm{X}}^j\sharp\widetilde{\mathrm{X}}^k}{2} + +m^2\tilde{t}E\sin\vartheta\frac{\partial\vartheta}{\partial\mathrm{X}^i} = 0. \tag{11}$$

To exemplify the temporal structure of a geodesic, an expanding interval $\mathrm{X}^0 = \langle\forall|\ \tau-\tau_0\rangle$ (read "no matter the scale of $\tau-\tau_0$") is a finite time-like path that holds an intrinsic stretching thermal energy and remains continually swelling[4] . So, as the soliton warps the space-time, we may interpret the $\widetilde{\mathrm{X}}^j$ as the transformation components coupled to the metric. Therefore, the terms in $\widetilde{\mathrm{X}}^j$ are the ones that head the description of the phenomenon, providing information about the way in which the metric undergoes inflection caused by the solitary wave. Also, to include the proposed semiology, we may define an operator $\Delta\sharp$ such that

$$\Delta\sharp\left[\zeta_{ij},\ \mathrm{X}^j\right] = \frac{d}{ds}\left(\zeta_{ij}\sharp\widetilde{\mathrm{X}}^j\right) - \frac{\partial\zeta_{jk}}{\partial\mathrm{X}^i}\sharp\frac{\widetilde{\mathrm{X}}^j\sharp\widetilde{\mathrm{X}}^k}{2}. \tag{12}$$

Lastly, equation 11 changes to

$$\Delta\sharp\left[\zeta_{ij},\ \mathrm{X}^j\right] + \frac{\partial\phi}{\partial\mathrm{X}^i}e^{2\phi}\tilde{t}^2 + m^2\tilde{t}E\sin\vartheta\frac{\partial\vartheta}{\partial\mathrm{X}^i} = 0, \tag{13}$$

and in the spatial infinity, assuming field ϕ asymptotically constant, $\lim_{r\to\infty}\phi = const.$, or even cancelled, and considering the limit of small ϑ, we can say that

$$\Delta\sharp\left[\zeta_{ij},\ \mathrm{X}^j\right] + m^2\vartheta\tilde{t}E\frac{\partial\vartheta}{\partial\mathrm{X}^i} - \frac{1}{6}\vartheta^3\tilde{t}E\frac{\partial\vartheta}{\partial\mathrm{X}^i} + ... = 0, \tag{14}$$

with remaining terms $\mathcal{O}\left(\vartheta^5\right)$ and higher. Details on the components $\widetilde{\mathrm{X}}^j$ are explained in reference [11]. Equation (13) is a synthetic and elegant way of displaying, on one hand, the bulk that shapes the phenomenon, on the other, the complements that stabilize it so that it is recognizable. Evidently, $\Delta\sharp$ is not a generic operator, but a deformation non-commutative operator of the metric under the conditions specified by the Lagrangian of the system.

Alcubierre metric and the shape function

A final example is illuminating (considerably more could be said but it is enough to the reader which will take examples in his own field of research). Inferring the shape function,

$$f_{(r_s)} = \frac{\sqrt{c^2 - e^{2\phi}}}{v_s}, \tag{15}$$

at the junction of a type-Alcubierre warp bubble with the external space-time, I explained that the null-diagonal metric field matrix ζ_{ij} has components ζ_{i4} of the form $-4E^{-1}\sqrt{1-e^{2\phi}}$, where E is the energy of space-time expansion (please, see reference [12] for an explanation considering the static metric $ds^2 = -e^{2\phi}dt^2 + \zeta_{ij}dx^i dx^j$). In this case, P-♯ indicates a fundamental mode operation on the inverse of the energy E at the junction, i.e.,

[4] This is the essence, the ontological foundation of the idea of continuity specifically in physics.

$-4E^{-1}\sharp\sqrt{1-e^{2\phi}}$. In other words, components ζ_{i4} of the metric field matrix,

$$\zeta_{lk} = \begin{bmatrix} 0 & 2E^{-1} & 2E^{-1} & -4E^{-1}\sharp\sqrt{1-e^{2\phi}} \\ 2E^{-1} & 0 & 2E^{-1} & -4E^{-1}\sharp\sqrt{1-e^{2\phi}} \\ 2E^{-1} & 2E^{-1} & 0 & -4E^{-1}\sharp\sqrt{1-e^{2\phi}} \\ 2E^{-1} & 2E^{-1} & 2E^{-1} & 0 \end{bmatrix}, \quad (16)$$

show that the expansion energy at the junction inflects the shape function (and, of course, how this inflection occurs).

Then, we have a representation of the way in which expansion energy — the dark energy, as I understand it — acts on the shape function in the matching between Alcubierre and static metrics, and this is a basal process to be identified, since expansion is intrinsic to space-time nature.

II: OTHER DIRECTIONS, SAME UNIVERSE

On the scale of the cosmos, only the fantastic has conditions to be true.

Teilhard de Chardin

While a physics student many years ago I heard a cliche common to theoretical disciplines: "...this is very important", or "...this is also very important", said the teacher pointing to some terms of an equation without much concern with the "why" of the importance. This is how obscure ideas propagate through generations; it is enough that we accept them once.

There is the same problem in all languages, namely, the question of clarity and accurate interpretation. The "why" of the importance of a given mathematical term crucial to ensure the purpose of the equation has to reflect a relevant bringing-of-an-action of thought for the understanding and for the consistency of the theory behind; the "why" has to do with the ultimate meaning of the equation. This leads us to always keep in mind, with Kim *et al.*, that

"[...]*learning physics equations should go beyond understanding mathematical relationships of the equations. In this sense, it is necessary to distinguish the difference between mathematical and physical aspect when learning the meaning of a physics equation. The mathematical meaning of a physics equation involves quantitative relation and mathematical operation among the symbols included in the equation.*" [7].

In cosmology, I think we are living in an era of dispersive fascination with the colossal amount of data that we now have at our disposal (add to this the loss of precious time consuming intellect with the invention of other universes, if we still can't explain the only one we know!). However, it is necessary to know in which theoretical framework we will work and interpret them "to what", thus reaching more fundamental conclusions. It seems to me that the more data we have, the more confused we get. This feeling alerts us to a key questioning upon what we are actually discovering and studying about the structure of the cosmos, its evolution and its origins. There is a whole propaedeutics to be reviewed, and this includes the possibility of semiological improvements. At least we have the advantage that general relativity turns out to be a sufficiently robust theory to support some external contributions that leave it less overcharged by the load of being asked to account for everything. It is not, therefore, a question of just seeking solutions to Einstein's equations (with very few exceptions, this has served to increase the perception of underdetermination in cosmology), but also of seeking to apply them from a broader and more theoretically elaborate semantics; deep down, it is much more than simply mathematics.

The lack of interest in the points I discuss is fueled by idealism — with its multiverse and inaccessible dimensions — and by what we could call "scientific corporatism" driven on behalf of motivations other than those of open, full and inclusive science. It is a fact that we are going through controversial situations in the way physical science has been conducted today. There are strong indications that the simplistic route has dominated scientific production in this area in recent times, blocking original initiatives that could very well shed light on a series of open questions. Academic environments seem more and more concerned with citation numbers and impact factors than with science itself. Furthermore, it would be very naive to imagine that science is immune to human ills, such as envy, vanity and selfishness.

In my personal opinion, I think that the way of doing theoretical physics — mostly in cosmology — needs to be revised. The universe is such that it is possible to represent it reasonably well with the help of mathematical formalism. This does not mean, as some would like, that the universe is mathematical. Science is best conveyed in English, which is not to say that it is English. It is the thoughtless uses of language that give rise to confusion and end up leading to meaningless inferences. I even believe that the adequate symbology can help the selection of better theories and models for the understanding of the cosmic structures and, virtually, for the future development of new technologies, including some kind of gravitational navigation that may improve our prospects for interstellar transport, of course, within the constraints imposed by known physics.

In our methodological revision, Leibniz's classic works, whether philosophical, mathematical or theological, must always be under our sights (so it goes for Kant!). More than his precocious and brilliant dissertation "On Combinatorial Art" (1666-1668), which aimed to create a method by which all the truths of reason would be reducible to a type of calculation [9], Leibniz planted the seed of a true mathematical linguistics that, very unlike being static, now allows us to elaborate an entire improved symbology to create representations in physics with greater semantic precision.

From all this discussion it is easy to conclude that languages, scales, grammars, metrics, none of that exists; they are just the means at our disposal to bring the incommensurability of the universe to the remarkable finitude of human understanding (from the cosmological point of view, I believe that a first step towards the very dematerialization of languages — and their postulated connections with reality — was taken with the introduction of structural invariance under scale changes, especially when dealing with Planckian and sub-Planckian dimensions [13, 14]). That is why there is not much epistemological future in thinking that the universe is mathematical, since it takes tremendous mental effort to make it fit our models. In particular, the scales we create are so that we can keep parts of the world within our cognizable purview in face of the true unfathomability of reality. So, for mathematics to play its role in the best possible way as a language for physics, it needs to be considered in ever-evolving symbolic characterization, adopting semantic and grammar enhancements whenever necessary. This can be done from simple and luminous ideas like Leibniz's.

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