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Reviewing Redshift and Cosmological Distances

The Sunyaev-Zel'dovich Effect and the LTB Perspective

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Abstract: Present article carries out an objective review of the Lemaître-Tolman-Bondi inhomogeneous cosmology, emphasizing its main theoretical and observational features, showing that up to this moment it is not possible to reject such model based on current data. The model is fine-tuned to type Ia supernovae data. Sunyaev-Zel'dovich effect is discussed in that cosmology.

Key words: type Ia supernovae, Lemaître-Tolman-Bondi cosmology, Sunyaev-Zel'dovich effect, redshift, unique geodesic.
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Nomenclature

B: the third LTB arbitrary function

c: LTB best fit constant

c: light speed

Greek letters

μ, ν : Greek indices ranging from 0 to 3

$\widehat{\Omega}$: Angular quantity in metric expressions

$g_{\mu\nu}$: Tensor metric

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1. Introduction

At small length scales it was observed deviations from the postulated homogeneity of the Universe at large scales, a fact that imposes the need to investigate whether the Friedman-Lemaître-Robertson-Walker (FLRW) metric is really adequate to describe the accelerated cosmological expansion.

The Lemaître-Tolman-Bondi (LTB) cosmology have been applied with some interesting results as an alternative to explain the universe without cosmological constant at scales $O(10)h^{-1}Mpc$ or even larger. In spite of the challenges it confronts and the objections faced to its major presuppositions, there is a general feeling that compels all of us to agree with the fact that it shed new lights on many critical questions, however we have to live together with more restrictive conditions accepting a null Λ. Nevertheless, the "trademark" of LTB models is to consider cosmologies in which the inhomogeneous universe is filled by non-rotational dust matter and not the null Λ itself; it is possible to work in LTB with not-null Λ and compare the results with the standard FLRW model.

In the past fifty or sixty years, philosophy and science came near in such a way that nobody may neglect the real gains for both sides. Nonetheless, in several situations philosophical discussions stay merely idealistic, with no effective contribution to clarify scientific dilemmas. That is the case of a common idealistic objection pointed out to LTB models: how anthropocentric is the placement of human observers in the center of the universe? To refuse the objection implicit in such question it is enough to remember that all we really have are the measurements we do here in the Earth, not in a "NGC"; we don't know what is going on there. Thus, to suppose to be in the center or not is a pseudo-dilemma to be solved bearing in mind only technical advantages. For LTB models it seems very interesting, from a theoretical view point, to put the Earth in the center of the universe. Besides, this question may be simply opposite by another that asks where the center of a cosmological spherical surface is.

The next objection, whether so we can say, is that with three arbitrary parameters the LTB model virtually may adjust itself to any observational data. This is a naïve criticism to say the least, and here philosophy comes to contribute. Observation and theory live in a dialectic balance, in which both are always talking with one another; observation paints a brain canvas with inexact images; theory tries to simulate the causal chain behind the painted canvas. They are very far from perfection, but they are constantly gauging each other. Theory looks for the best fitting to observational data and chances some predictions; observation supplies the realistic foundations to discuss theory and to construct models, but also accepts "orders" from theory (predictions). So, the scientific question is not how much parameters are needed to that conversation between theory and observation, but how large is the data best covered by the theory, how much distinct sources of observational data are well gauged by the theory at the same time. What we expect from a LTB model is its simultaneous and reasonable concordance with data from microwave background, from SN Ia, from structure formation and so forth. Faced with the ontological and inevitable lack of complete isomorphism between model and reality, we cannot be so radical in criticize theories which, in resume, do what is possible to do.

Let me do some considerations about the fundamental choice of the present proposal:

a) I shall be treating the situation in which there is a unique geodesic, so I do not deal *a priori* with evolution.
b) As pointed out by Brouzakis *et al.* [1], the LTB metric can reproduce any relation between luminosity distance and redshift, which signifies that type Ia supernovae data may proceed from an inhomogeneous distribution of matter having ground in that metric. In this

investigation I precisely shall describe, by the connection between geometry (curvature) and cosmological redshift, a representation of the luminosity distance as direct consequence of the LTB inhomogeneous context.

2. The standard approach

2-1. A bit of history

At 1917, shorter there after Einstein's announcement of his cosmological model, it appeared a seminal paper published by the Dutch astronomer de Sitter in which he showed that an empty universe need not have the metric of Minkowski, usually considered as the limit of the relativistic metric at large distances of all gravitating matter [2]. Connecting cosmological constant with commoving radius, de Sitter world-line element assumes the form

$$ds^2 = -\frac{dr^2}{1-\frac{r^2}{R^2}} - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 + \left(1-\frac{r^2}{R^2}\right)dt^2,$$

where the constant curvature is $\frac{1}{R^2} = \frac{\Lambda}{3}$.

The most interesting feature I emphasize here is that Schwarzschild's exterior solution,

$$ds^2 = -\frac{dr^2}{1-\frac{2m}{r}-\frac{\Lambda r^2}{3}} - r^2 d\theta^2 - $$
$$-r^2 \sin^2\theta d\phi^2 + \left(1-\frac{2m}{r}-\frac{\Lambda r^2}{3}\right)dt^2$$

for a static field in the empty space surrounding a massive sphere, falls into de Sitter world-line element if the mass of the sphere tends to zero at the origin. Tolman referred to this last equation as a very important example of a cosmological line element related to an inhomogeneous model [3], even though today it is clear that at large scales Schwarzschild geometry is not proper to deal with inhomogeneities.

Generalizing, we may put m=0, which is the same thing to suppose an universe completely empty, so that

$$ds^2 = -\frac{dr^2}{1-\frac{\Lambda r^2}{3}} - r^2 d\theta^2 - r^2 \sin^2\theta d\phi^2 + \left(1-\frac{\Lambda r^2}{3}\right)dt^2 \;\therefore$$

$$ds^2 = -\left(1-\frac{\Lambda r^2}{3}\right)dt^2 + \frac{dr^2}{1-\frac{\Lambda r^2}{3}} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right).$$

2-2. The contemporary understanding of inhomogeneous cosmology

Although technology has undergone remarkable advances from Lemaître to now, the limitations in our observational tools are still, crudely speaking, very real. It is difficult to imagine in what degree those limitations shall be minimized during the next thirty/fifty years. Gaps of information about the true content of the universe have been diminished, thanks to the great effort of scientific community, but many crucial points remain unsolved even regarding the good approximations of ΛCDM representations. In this scenario, LTB cosmology emerges as a natural option not only to try to account for the apparent cosmic acceleration with no Λ, but to model a universe beyond appearances.

I begin writing Einstein's field equation ($\Lambda g_{\mu\nu}$ is placed at the left-hand side since I always emphasize possible changes in geometry),

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{M_{Pl}^2} T_{\mu\nu} \,,$$

which reduces to

$$G_{\mu\nu} = \frac{1}{M_{Pl}^2} T_{\mu\nu} = \frac{1}{M_{Pl}^2} \rho u_\mu u_\nu \qquad (1)$$

for null Λ. Here, $M_{Pl} = 1/\sqrt{8\pi G_N}$ (reduced Planck mass), ρ is the energy density and u^μ the speed of the matter flow, such that $u_\mu u^\mu$ = -1. Spherical symmetry leads to an exact solution of equation (1).

The standard LTB metric space provides a geometry embedded in a supposed universe inhomogeneously filled by a pressure-negligible dust matter. It must be clear that this supposition is not perturbatively related to any FRW cosmology. The associated world-line element is given by

$$ds^2 = -dt^2 + \frac{R'(r,t)^2\, dr^2}{1+f(r)} + R(r,t)^2\, d\hat{\Omega}^2 \,,$$

with coordinates $x^{i \,\equiv\, \{1,2,3\}} \equiv \{r,\theta,\phi\}$ synchronous commoving with matter (or $dx^i/dt = 0$). Cosmological constant $\Lambda = 0$; $f(r)$ is an arbitrary function only explicit in r. In a classical sense, functions $R(r,t)$ and $f(r)$ are related in accordance to Einstein's equations [4] by

$$\dot{R}(r,t)^2 = \frac{1}{M_{Pl}^2} \frac{m(r)}{R(r,t)} + f(r) \,; \qquad (2)$$

$$\rho(r,t) = \frac{m'(r)}{R(r,t)^2 R'(r,t)} \,, \qquad (3)$$

where $m(r)$ denotes an arbitrary function that describes the energy (gravitational baryonic mass) contained inside commoving radius r. It is worthwhile to comment that some authors emphasize the model focusing mass, some focusing curvature; nowadays it is a matter of personal taste, and I shall develop formal considerations on curvature, since my focus is on the theoretical behavior of the model when we associate explicitly geometry and redshift in LTB cosmology.

Geodesic equation $\xi^\nu \nabla_\nu \xi^\mu = 0$ takes the form

$$\frac{d^2 t}{dv^2} + \frac{\dot{R}' R'}{1+f} \left(\frac{dr}{dv} \right)^2 + \dot{R} R \mathfrak{L}^2 = 0 \,;$$

$$\frac{d^2 t}{dv^2} + 2 \frac{\dot{R}'}{R'} \frac{dt}{dv} \frac{dr}{dv} \left(\frac{R''}{R'} - \frac{f'}{2(1+f)} \right) \left(\frac{dr}{dv} \right)^2 - (1+f) \frac{R}{R'} \mathfrak{L}^2 = 0,$$

$$\frac{d^2 \theta}{dv^2} + 2 \frac{\dot{R}}{R} \frac{d\theta}{dv} \frac{dt}{dv} + 2 \frac{R'}{R} \frac{dr}{dv} \frac{d\theta}{dv} - \sin\theta \cos\theta \left(\frac{d\phi}{dv} \right)^2 = 0,$$

$$\frac{d^2 \phi}{dv^2} + 2 \frac{\dot{R}}{R} \frac{d\phi}{dv} \frac{dt}{dv} + 2 \frac{R'}{R} \frac{dr}{dv} \frac{d\phi}{dv} - 2 \cot\theta \frac{d\theta}{dv} \frac{d\phi}{dv} = 0,$$

where

$$\mathfrak{L}^2 \equiv \left(\frac{d\theta}{ds} \right)^2 + sin^2\theta \left(\frac{d\phi}{ds} \right)^2 .$$

The integration of the first equation gives

$$-\left(\frac{dt}{dv} \right)^2 + \frac{R'^2}{1+f} \left(\frac{dr}{dv} \right)^2 + R^2 \mathfrak{L}^2 = 0. \qquad (4)$$

Because of the positive-definite quantities of the two last terms in the above equation[1], to fulfill

[1] The arbitrary function f is assumed to be among the following three cases: $f = 0$; $f > 0$; $0 > f > -1$.

the geodesic null condition implies the negative sign of the first term.

From LTB standard approach, redshift is given by

$$In(1+z)=\int_0^{r_e}\frac{\dot{R}'(r,t)}{\sqrt{1+f}}dr \qquad (5)$$

for a receiver at $r = 0$, collecting signals from a emitter source at $r = r_e$.

3. The unique geodesic and the luminosity distance

On one hand, according to differential representation, light rays that are traveling radially ($d\theta^2+\sin^2\theta d\phi^2=0$) and moving inward follow the null geodesic

$$dt=-\frac{R'(r,t)}{\sqrt{1+f}}dr\ , \qquad (6)$$

From here we may say that

$$dr=-\frac{\sqrt{1+f}}{R'(r,t)}dt\ . \qquad (7)$$

Inserting into equation (5), it follows

$$In(1+z)=\int_0^{r_e}\frac{\dot{R}'(r,t)}{\sqrt{1+f}}dr=\int_0^{t_e}-\frac{\dot{R}'(r,t)}{\sqrt{1+f}}\left(\frac{\sqrt{1+f}}{R'(r,t)}\right)dt\ . \qquad (8)$$

Once that we have expression (7), there is no mathematical impediment to put

$$\int_0^{t_e}-\frac{\dot{R}'(r,t)}{\sqrt{1+f}}\left(\frac{\sqrt{1+f}}{R'(r,t)}\right)dt= \qquad (9)$$
$$-\int_0^{t_e}\left[\frac{d}{dt}\left(\frac{R'(r,t)}{(1+f)^{1/2}}\right)-\left(-\frac{1}{2}\frac{R'(r,t)\dot{f}}{(1+f)^{3/2}}\right)\right]\left(\frac{\sqrt{1+f}}{R'(r,t)}\right)dt.$$

As exposed previously, f is not an explicit function of t, but time and space are very entangled in relativity to discard *ad hoc* a profound physical analysis of the implications that function f can bring as implicit function of time. Besides, there is no loss of generality in equation (9), since we preserve the initial form at the left-hand side if

$$\dot{f}=0\ .$$

On the observer's past null cone, coordinate r may be chosen so that holds the gauge

$$\frac{R'(r)}{\sqrt{1+f}}=1\ [5].$$

We can now rearrange equation (9), introducing this choice and doing

$$In(1+z)=-\int_0^{t_e}\left[\frac{d}{dt}\left(\overbrace{\frac{R'(r)}{(1+f)^{1/2}}}^{=1}\right)-\left(-\frac{1}{2}\frac{R'(r)\dot{f}}{(1+f)^{3/2}}\right)\right]\left(\frac{\sqrt{1+f}}{R'(r)}\right)dt=$$

$$-\int_0^{t_e}\left(\frac{1}{2}\frac{R'(r)\dot{f}}{(1+f)^{3/2}}\right)\left(\frac{\sqrt{1+f}}{R'(r)}\right)dt=$$

$$-\int_0^{t_e}\frac{1}{2}\frac{\cancel{R'}(r)\dot{f}}{(1+f)^{3/2}}\frac{\sqrt{1+f}}{\cancel{R'}(r)}dt=$$

$$-\int_0^{t_e}\frac{1}{2}\frac{\sqrt{1+f}\,\dot{f}}{(1+f)^{3/2}}dt=$$

$$-\int_0^{t_e}\frac{1}{2}\frac{\dot{f}}{(1+f)}dt\ . \qquad (10)$$

In a certain manner, this procedure makes possible to maintain evolution after we have fixed unique geodesic and renders the problem more attractive from the point of view of the cosmological redshift.

From expression (10) I write luminosity distance as

$$DL = R(r)e^{-1/2\int_0^{t_e}\frac{\dot{f}}{(1+f)}dt}. \qquad (11)$$

Now, the distance modulus, a direct measure of the luminosity distance, is defined in Riess *et al.* [6] as

$$\mu = 5\log_{10}\left(\frac{DL}{1Mpc}\right) + \mathbf{25,}$$

but I rewrite this formula in a more general manner [7],

$$\mu = 5\log_{10}\left(\frac{DL}{1Mpc}\right) + \mathcal{M},$$

where $\mathcal{M}$ is a constant to be found in accordance to the fitting with an FRW model. Inserting expression (11), it comes

$$\mu = 5\log_{10}\left(\frac{R(r)e^{-1/2\int_0^{t_e}\frac{\dot{f}}{(1+f)}dt}}{1Mpc}\right) + \mathcal{M}.$$

Trivial integration gives

$$DL = 5\log_{10}\left(\frac{R(r)e^{-1/2 ln(1+f)|_0^{t_e}}}{1Mpc}\right) + \mathcal{M}. \qquad (12)$$

4. The LTB model applied to type Ia SN data

Now I introduce the following three quantities as in reference [7]:

$$R(r,t) = ar,$$
$$f = Ar^2,$$
$$F = Br^3,$$

where a is the scale factor, A and B are two LTB arbitrary functions to be defined. We relate a, A and B by the differential equation

$$\dot{a}^2 = A + \frac{B}{a}. \qquad (13)$$

The $A = 0$ FRW cosmology assumes the standard form $a = t^{2/3}$ for $B = 4/9$. To verify it is enough to get the solution of equation (13) for $t_0 = 0$,

$$t - t_0 = \int_0^a \frac{du}{\sqrt{A + \frac{B}{u}}}$$
$$= \int_0^a \frac{du}{2/3\sqrt{u}}$$
$$= \frac{3}{2}\int_0^a u^{1/2}du$$
$$= \frac{3}{2}\frac{u^{3/2}}{3/2}$$
$$= a^{3/2}.$$

We note that, due to coordinate freedom to deal with *r*, *B* would be also a function which converges to 4/9 at the infinite. In the LTB model, accordingly the choice of *A* and *B*,

$$\Omega_M = \left(1 + \frac{a}{B}\right)^{-1} \therefore$$
$$1 + \frac{a}{B} = \Omega_M^{-1} \therefore$$
$$a = B\left(\Omega_M^{-1} - 1\right).$$

Function *A* has the form

$$A = \frac{1}{1+(cr)^2} \quad [7],$$

with c being a best fit constant.

At last, according to Garfinkle (private communication), it was used the "effective magnitude" of Perlmutter *et al.* [8] defined as an "effective rest-frame B magnitude". Remaining mismatches of wavelength coverage were corrected by the "cross-filter *K*-correction". The simulations were performed for a model with parameters Ω_M and c=8.5. The value of c was chosen for best fit with supernova data.

Figure 1 exhibits the level of concordance between supernovae data (with error bars) and the proposed model. Figures 2 and 3 show the observational luminosity distance and check it with present model.

5. The Sunyaev-Zel'dovich effect in LTB model

Rashid Sunyaev and Yakov Zel'dovich [12] predicted that during the path at the vicinity of a galaxy cluster some photons of the cosmic microwave background (CMB) perceive Compton scattering of hot electrons inside the cluster. The distortions (referred to that scattering) produced in the black-body spectrum of the CMB settle the so called Sunyaev-Zel'dovich (SZ) effect. If the cluster has a peculiar velocity (non-zero velocity along the watching line), there is a kinetic SZ and the study of its watching line component may give information about the motion of the cluster. The peculiar velocity (radial) is given by $Vr = cz - rH_0$, where z is the cluster redshift and r its distance.

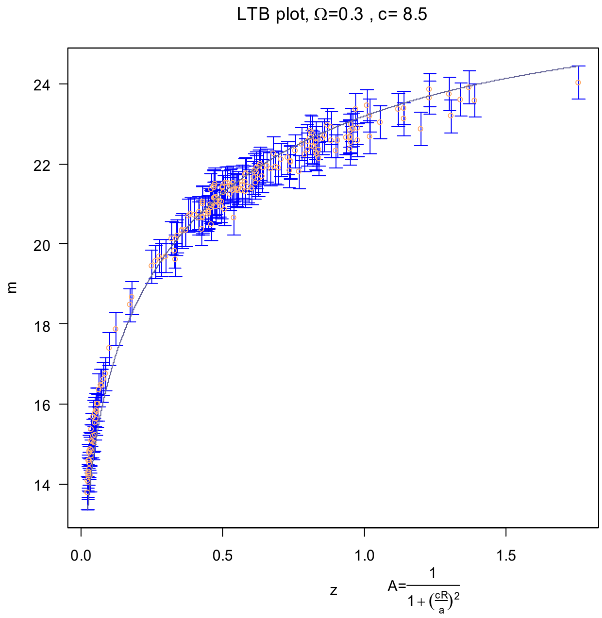

Figure 1. Plot of effective magnitude *versus* redshift and supernova data from Riess tables (gold accurate).

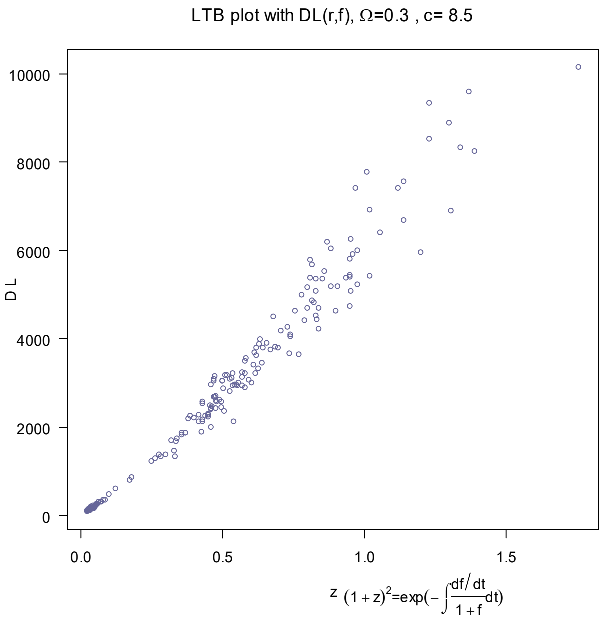

Figure 2. Plot of the supernova data (*DL versus* redshift) from Riess tables (gold accurate).

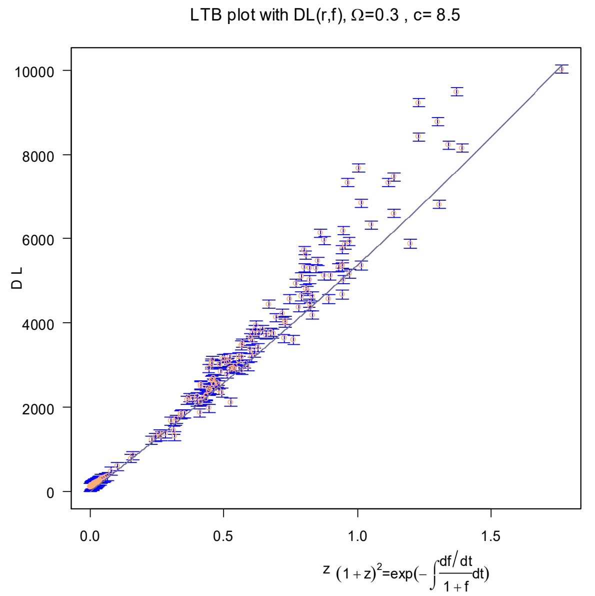

Figure 3. Plot of *DL versus* redshift (equation (12)) and the supernova data from Riess tables (gold accurate).

For nearby clusters ($z << 0.2$), X-ray observations combined with the intensity variation including SZ effect allow to evaluate the FRW angular diameter distance D_A in terms of redshift, deceleration parameter and Hubble constant. If $\Lambda = 0$,

$$D_A = \frac{c}{H_0 q_0^2}\left(\frac{q_0 z + (q_0 - 1)\left(\sqrt{1+2q_0 z} - 1\right)}{1+z^2}\right). \quad (14)$$

The distances to galaxy clusters were determined from joint analysis of X-ray observations and of 30 GHz interferometric SZE observations [16]. For comparative purposes, we may refer to a spherical isothermal model (the β model) by the general expression of the SZE angular diameter distance

$$D_A = \frac{(\Delta T_0)^2}{S_{x0}}\left(\frac{m_e c^2}{k_B T_{e0}}\right)^2 \frac{\Lambda_{eH0}\mu_e/\mu_H}{4\pi f^2_{(x,T_e)} T^2_{CMB}\sigma_T^2 (1+z)^4} \times$$

$$\times \frac{1}{\theta_c} \frac{\int \left(\frac{n_e}{n_{e0}}\right)^2 \frac{\Lambda_{eH}}{\Lambda_{eH0}} d\eta \,|_{R=0}}{\left[\int \frac{n_e}{n_{e0}} \frac{T_e}{T_{e0}} d\eta \,|_{R=0}\right]^2}, \quad (15)$$

where ΔT_0 is the central thermodynamic SZE temperature decrement (or increment), $T_{CMB} = 2.728$ °K is the temperature of the CMB, Λ_{eH0} is the so called "X-ray cooling function"[2] in *cgs* units (Λ_{eH} is the same but taken at the cluster rest frame), $f_{(x,T_e)}$ is the frequency dependence of the SZE with $x = h\nu / kT_{CMB}$, σ_T is the Thompson cross section, S_{x0} is the normalization of the X-ray surface brightness S_x used in the fit, θ_c is the characteristic angular scale of the galaxy cluster, n_e is the electron number density, n_{e0} is the cluster central density, η is the line-of-sight length in units of the characteristic radius $r_c = \theta_c D_A$ (see [16, 17] for more details).

In fact, this way to calculate distances currently involves high uncertainties because

[2] The cooling function enters basically as the conversion between detector counts and cgs units.

rarely clusters exhibit spherical symmetry in the distribution of the gas. Nevertheless, it is accepted that any important bias is introduced in the Hubble parameter inferred from a large sample of clusters.

5.1. The contemporary understanding of inhomogeneous cosmology

Now we can imagine a cosmological LTB <<bubble>> embedded in a FLRW background, in such a way that we can ask whether the angular diameter distance measured from the background would be the same measured from the bubble. This question makes sense, since it would be careless to neglect the effects of inhomogeneity on distance measurements.

In Figure 4, I classified 38 clusters [13,14,20], according to the Chandra X-ray Observatory[3], by dissimilarities in the pair of variables (D_A, z), constructing 6 fiduciary mega-clusters adopting for each one the medians of z and of D_A, as well as the error averages in D_A. In the simulations, it was noteworthy that the curvature of D_A is sensitive to small changes in the gauge parameter Γ. To get an idea, by adopting the natural gauge ($\Gamma = 1$), the curve referring to the observer centered on the cavity LTB becomes very close to the curve in FLRW. In the plot, the respective theoretical curves defined from inside the LTB cavity[4] (orange line) and from external FLRW background (green line) were superposed. Error bars include statistical uncertainties intrinsic to X-ray observations and SZ effect measurements.

At the specified conditions, the curve referring to the observation taken from the LTB bubble shows a better adjustment to the observational data. The uncertainties in the registration of angular diameter distance are such that it would be, to say the least, arbitrary any definite conclusion based on such a precarious observational conjuncture.

6. Conclusions

The LTB model discussed here showed high potential to explain simultaneously data from type Ia SN and from galaxy clusters, including SZ effect in the latest.

In different approaches, time derivative of the curvature was recently treated in a perturbative context on large scales [10] and in interesting discussions on Two Measures Field Theory (TMT) in the context of spatially flat FRW cosmology [11]. Also less orthodox approaches has been defended considering variable deceleration parameter models [15]. According to my view, I claim that curvature time derivative provides an additional trace of high inhomogeneity at large scales and it is free from singularities as $f = 0$; $f > 0$; $0 > f > -1$. Intuitively, df/dt would be negligible at small scales. Besides, its introduction in the model partially rescues evolution lost with the choice of the unique geodesic. Luminosity distance, rewritten as an explicit function of the curvature, is interpreted as a possible theoretical alternative to account for the dimming of distant type Ia SN apparent luminosity.

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[3] Indeed, there are now better mastered new catalogs extracted from the larger SDSS survey, in particular the catalog redMaPPer [18]. Also, the cluster catalogs extracted from the Planck Collaboration, SPT and ACT data also increased the number of known clusters [19].

[4] The calculation principles for adjusting the angular diameter distance as a function of inhomogeneity are the subject for another publication.

In short, time variable function f resumes three fundamental features: a) the redshift is taken as a function of the curvature and its time derivative (for large radii). For small radii, it is enough the relativistic approximation (r = const.); b) redshift entangled with curvature may be seen as another tool to "observe" curvature itself, and time differentiable curvature is a reasonable supposition at large scales for the inhomogeneous expanding universe; c) theoretical LTB-SZE line D_A *versus* z tends to have a curvature more open than the well known FRW-SZE curve, which means that larger angular diameter distances at high redshifts would be best represented by the former.

As I pointed out in the beginning of this article, to validate LTB spacetime as a viable model it is necessary to perform more simulations with different cosmological data. I am just working to test the compatibility of the model also with data from galaxy counts.

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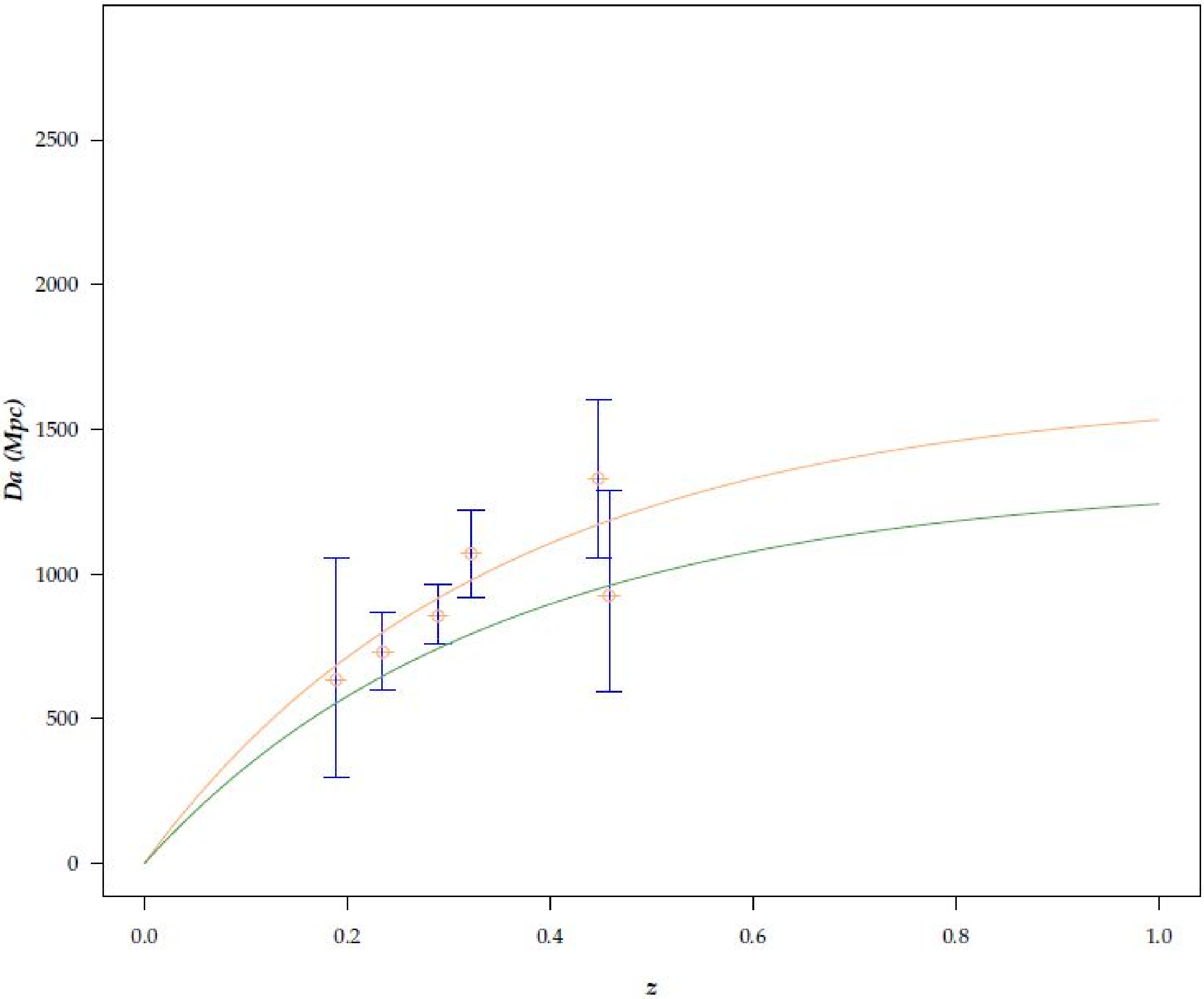

Figure 4. Fiduciary mega-clusters constructed from the dissimilarities of the 38 elementary clusters calculated on the pair of variables (D_A, z) and organized by the method of *partition by medoids*, $\Gamma = 1.19$. The matrix of dissimilarities computed the roots of the sums of the squares of the differences between the elementary clusters. The reader must note the degree of fit of the curve defined from within the LTB cavity (in orange) compared to the FLRW curve (in green).

Appendix

We may prove that the introduction of *df/dt* is consistent with the choice of large scale observations. Let us take the LTB arbitrary function defined previously,

$$f = Ar^2 = \frac{r^2}{1+(\mathrm{c}r)^2}.$$

First of all, we note that the time derivative is given by

$$\dot{f} = \frac{2r\dot{r}}{1+c^2r^2} - \frac{2c^2r^3\dot{r}}{(1+c^2r^2)^2}.$$

By comparison, if $r << 1$, the second term of the right-hand side is much smaller than the prime. So, for small *r*,

$$\dot{f} = \frac{2r\dot{r}}{1+c^2r^2}$$

is a good approximation.